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UNITED STATES NAVAL POSTGRADUATE SCHOOL



THESIS

A METHOD OF INCORPORATING BEND FLEXIBILITY FACTORS
IN MODELS FOR PIPING FLEXIBILITY ANALYSIS

Peter H. Freeman

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A METHOD OF INCORPORATING END FLEXIBILITY FACTORS
IN MODELS FOR PIPING FLEXIBILITY ANALYSIS

by

Peter H. Freeman

Lieutenant, United States Navy

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
MECHANICAL ENGINEERING

United States Naval Postgraduate School
Monterey, California

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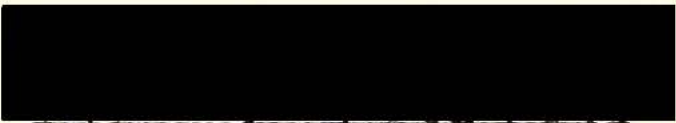
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the thesis requirements for the degree of
MASTER OF SCIENCE

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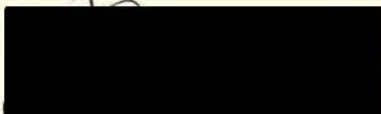
MECHANICAL ENGINEERING

from the

United States Naval Postgraduate School


Chairman
Dept. of Mechanical Engineering

Approved:


Academic Dean

ABSTRACT

Analysis of piping flexibility by means of tests with models is undertaken both independently and as an adjunct to analytic or graphical methods of analysis. One difficulty encountered in model-test methods is the problem of incorporating in curved portions of the model the extra bending flexibility encountered in curved pipe.

This paper describes an attempt made to solve the problem by the use of very thin-walled tubing as a model in an already existing apparatus which permits determination of reactions by measurements of deflections in the model. Although the attempt was unsuccessful because the tubing was unknowingly loaded beyond its yield point, it is concluded that the basic idea does hold some promise for moderate values of bend flexibility.

The assistance and encouragement of Professor John E. Brook of the faculty of the U. S. Naval Postgraduate School, Monterey, California, is gratefully acknowledged.

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TABLE OF SYMBOLS

A	Extremity of pipe or model opposite the extremity at the origin of the stated problem.
d	Mean diameter of pipe or tube in inches.
d _i	Inside diameter of pipe or tube in inches.
d _o	Outside diameter of pipe, tube or rod in inches.
D ₂	Translation in inches in y" direction of point P.
D ₅	Translation in inches in z" direction of point P.
D ₄	Angular displacement in radians about x" axis measured at P.
D ₅	Angular displacement in radians about y" axis measured at P.
D ₆	Angular displacement in radians about z" axis measured at P.
E	Modulus of elasticity in pounds per square inch.
F	Force in pounds acting on structure; see Table of Subscripts.
$\bar{F}$	Resultant force in pounds acting on structure.
F ₂	Shear force in pounds at A in y' direction.
F ₅	Shear force in pounds at A in z' direction.
F ₄	Moment in pound-inches at A about x' axis.
F ₅	Moment in pound-inches at A about y' axis.
F ₆	Moment in pound-inches at A about z' axis.
g	$2(1 - \nu) r^2$ in inches ²
G	Modulus of elasticity in shear in pounds per square inch.
I	Moment of inertia of cross section with respect to neutral axis in inches ⁴ .
J	Polar moment of inertia in inches ⁴ .
k	Bond flexibility factor (See text, page 3). Dimensionless.
l ₀	Free length of a spring in inches.
L	Linear distance in inches from A to P.

M	Moment in inch-pounds acting on structure; see Table of Subscripts.
$\bar{M}$	Resultant moment in inch-pounds acting on structure.
n	Number of turns in a spring.
O	Point of origin of problem as originally stated, and origin of the unprimed system of coordinates.
P	Point of measurement of deflections; found at longitudinal center of mirror-mounting barrel.
Q	First moment of area of half of cross-section with respect to neutral axis in inches ³ .
r	Mean radius of pipe or tube in inches.
$\bar{r}$	Mean radius of a coil of a spring in inches.
r_i	Inside radius of pipe or tube in inches.
r_o	Outside radius of a pipe or tube in inches.
R	Radius of bend in inches.
s_n	Normal stress in pounds per square inch.
s_s	Shear stress in pounds per square inch.
t	Wall thickness in inches.
X, Y, Z	$x, y,$ and z coordinates of A.
x', y', z'	Coordinates with origin at anchor with directions same as x'', y'' and z'' respectively. See Fig. 4.
x, y, z	Coordinates of a general point taken with respect to origin at O as stated in original problem. See Fig. 4.
x'', y'', z''	Coordinates with origin at P . See Fig. 4.
x''', y''', z'''	See Appendix II.
α	Angle of inclination in radians of any coil of a spring with a circular section of the cylinder on which the coil lies.
β	Flexural rigidity in pounds-inches ² .
γ	Angle between a radius line of a cross-section and the horizontal, measured counter-clockwise in radians.

δ	Deflection due to shear in inches.
Δ_m/Δ_p	Ratio of deformations applied in model to prototype deformation, dimensionless.
θ	Slope of shear deflection curve in radians.
μ	Poisson's ratio, dimensionless.
ξ, η, ζ	Coordinates of a reflected point of light on cross section paper for rotation measurement, such that $\xi_2 = \xi_1, \eta_2 = \eta_1$, and $\zeta_2 = \zeta_1$ indicate angular displacements about x'', y'' , and z'' , respectively in millimeters.
T	Torsional rigidity in pounds-inches ² .

TABLE OF SUBSCRIPTS

A	Refers to point A.
m	Refers to model.
O	Refers to point O
p	Refers to prototype (actual piping system).
x,y,z,	Refer to components in directions x,y, and z, respectively.
x',y',z'	Refer to components in directions x', y', and z', respectively.
x",y",z"	Refer to components in directions x", y", and z", respectively.
1	Pertains to set of data taken with system deformed in negative direction. This does not apply to use of numbered subscripts with D or F.
2	Pertains to set of data taken with system deformed in positive direction. This does not apply to use of numbered subscripts with D or F.

1. Introduction

For design purposes the desirability of some simple system for predicting the end reactions in a variety of proposed piping systems subjected to thermal expansions is quite evident. A great many schemes have been suggested and used. In general they all fall into one of three categories: analytical, graphical, and mechanical (model-test). Model-test systems generally employ equipment capable of causing the ends of a model of the proposed system to move relative to one another an amount proportional to the extensions which would be found in the heated prototype if unrestrained. End reactions are found either by direct force-measurements or by inference from measurements of deflections at some point within the model. This paper is concerned with a model-test system employing a deflection-measuring apparatus constructed by Charles F. Rauch, Jr. [1] at the United States Naval Postgraduate School, Monterey, California, in 1957.

A complicating factor in analyzing the behavior of pipe subjected to forces at the end supports is the fact that wherever the pipe changes direction its behavior may be different from that in straight sections. Specifically, in bends and elbows the torsional rigidity remains the same as in straight sections, but the flexural rigidity often decreases. The ratio of flexural rigidity in straight pipe to that in a particular bend or elbow in the same pipe is called the flexibility factor, k , of the bend and can be as high as 50 in extreme cases with thin-wall sharply curved elbows. S. Timoshenko presents [2] a description of the cause of decreased flexural rigidity in bends and a derivation of the expression for flexibility factor, k . In general terms, what happens is that under bending the circular cross-section flattens into an ellipse, producing an effect equivalent to a

reduction in the moment of inertia. For bending moments in the plane of the bend the major axis of the ellipse is normal to the plane of the bend, and for out-of-plane bending it is inclined at some other angle. The resultant decrease in rigidity is the same, however, regardless of whether the bending is in- or out-of-plane. The effect appears to be independent of internal pressure, but is considerably modified by the presence of any flanges or stiffening rings [1] .

2. Purpose of this Work

In his tests Rauch used models constructed of solid rod and chose to neglect bend flexibility factors or to analyze systems where the factors were unity. The purpose of the work described herein was to extend his method to include these effects. Thus a closer examination of bend flexibility is in order.

Various expressions for bend flexibility factors k have been proposed, but A. R. C. Markl [1] suggests that the following simplified form is entirely adequate for most purposes:

$$k = \frac{1.65 r^2}{tR}, \text{ where:} \quad (1)$$

r is the mean radius of the undeformed pipe,

t is the wall thickness, and

R is the radius of the bend.

This formula is incorporated in the code for pressure piping ASA 1331.1 - 1955.

In straight pipe the ratio of torsional to flexural rigidity is $\frac{1}{1+\mu}$, where μ is Poisson's ratio. In bends this ratio becomes $\frac{k}{1+\mu}$. The problem here, then, was to devise some means of reducing the flexural rigidity from EI to EI/k in the bends of a model while keeping the torsional rigidity constant throughout the model. At the same time it was desired that the method used be in keeping with the basic simplicity of Rauch's apparatus. Various ideas were considered before the one used was finally adopted. Thought was given to some scheme for merely modifying the cross-section of the bends in a solid rod model, but this would not work because a solid circular cross-section provides the maximum

ratio of torsional to flexural rigidity, and any change in the shape of the section serves to decrease rather than to increase the ratio. However, it should be noted in passing that for single-plane configurations, where no torsion is involved, one should be able to produce increased flexibility in bends merely by reducing the moment of inertia of the rod by the factor k .

Another idea considered was based on an article by J. D. Conrad [3] in which he described a method of increasing bend flexibility by using fairly thick-walled tubing for models and cutting slots in the bends. Correspondence with Mr. Conrad, however, revealed that he had no analytical method of pre-determining the slot depth or distribution, but that, instead, a "cut and try" approach was used. Such a procedure did not appeal to the present investigator.

It was next thought that perhaps a coil spring might be inserted in the place of the rod in the bends. However, an analysis (Appendix III) based on Wahl's [6] and Prescott's [7] spring equations showed that the maximum flexibility factor that could be simulated in this way would be 1.50; it was decided that this gave a range too restricted for the present investigation.

Consideration was also given to the possibility of constructing a model along the lines of a simplified analytical method proposed by Markl [5], but it developed that of the three simplifying assumptions necessary to his method only one could be incorporated in a rod model.

Correspondence from Lyle C. Andrews, formerly in charge of the M. W. Kellogg Company testing laboratory, states that he has invented a device for simulating bend flexibility but is loathe to divulge any details at the present time.

A practical and simple approach to the simulation of flexibility factors of moderate values appeared to lie in the use of tubing for models, and henceforward attention was restricted to problems involving such moderate values. Literature on the subject of model-testing and correspondence with other investigators indicated that tubing with sufficiently small wall thickness for the purpose was not readily available, but this turned out to be not strictly true. As will be seen later, the difficulty lay not in obtaining satisfactory tubing but in bending it. Let us see what would be required in a tube. Forces and moments at the ends of the model are to be calculated by measuring deformations at some point in the straight section at one end and using cantilever beam formulae. Scale factors relating forces and moments calculated in the model to those in the prototype are [9] :

$$\frac{F_p}{F_m} = \frac{E_p}{E_m} \frac{I_p}{I_m} \frac{\Delta_m}{\Delta_p} \frac{\lambda_m^3}{\lambda_p^3} \quad (2)$$

$$\frac{M_p}{M_m} = \frac{E_p}{E_m} \frac{I_p}{I_m} \frac{\Delta_m}{\Delta_p} \frac{\lambda_m^2}{\lambda_p^2} = \frac{F_p}{F_m} \frac{\lambda_p}{\lambda_m} \quad (3)$$

$\frac{\Delta_m}{\Delta_p}$ need only be large enough so that an adequate cantilever length, L , be provided for measurement of deflections and small enough so that the model will fit within the existing framework of the apparatus. These were the only limitations posed when rod was used for a model, but the use of tubing makes a further demand that the scale not produce model bend radii smaller than the practical limit for the tube size. Finally introduction of the requirement that the bend flexibility factors of model and prototype be the same still further restricts the choice of $\frac{\Delta_m}{\Delta_p}$ as follows:

$$\begin{aligned}
\left(\frac{1.35r^2}{tR} \right)_m &= \left(\frac{1.35r^2}{tP} \right)_p \\
\left(\frac{d^2}{t} \right)_m &= \frac{R_m}{R_p} \left(\frac{d^2}{t} \right)_p \\
\left(\frac{d^2}{t} \right)_m &= \frac{\lambda_m}{\lambda_p} \left(\frac{d^2}{t} \right)_p \quad (4)
\end{aligned}$$

Thus it can be seen that the linear and cross-sectional scales are interdependent. Whether or not they can be equal to each other depends on the prototype dimensions and the dimensions of available tubing. However, other than for the esthetic value there is no particular reason to make them the same.

3. Apparatus

As mentioned previously, Rauch's apparatus, which is carefully described in his thesis [8], was used for the tests conducted by the present investigator. Only a few modifications were found necessary to adapt it to the use of tubing instead of rod. The face plate of the milling head was removed, and in its place was installed an adapter manufactured for the purpose. (Fig. 1). Two identical end-plates (Fig. 1) were manufactured to accept the ends of the tubing. Note that these plates could be used for any size tubing merely by boring the central hole to the proper diameter. A new anchor plate was manufactured, as was a mirror-mounting barrel of the proper size. (Fig. 2). Finally larger bakelite knife edge supports were cut out and installed.

4. Selection of Scale

This is necessarily a trial-and-error process. The problem selected for test is shown in Fig. 4. First it was decided that a minimum cantilever length, L , (between the anchor and the point of measurement, P) of eight inches would be acceptable. With end sections of around 120 inches this meant a maximum length scale of about 15:1. Inserting this value, together with the given dimensions of the pipe, into equation (4) gave a value of 44.6 for $\left(\frac{d^2}{t_m}\right)$. Examination of the catalogs of several manufacturers of tubing indicated that the smallest tubing with approximately this value of $\frac{d^2}{t}$ had an outside diameter of 1.25 inches and a wall thickness of .022 inch, for which $\frac{d^2}{t}$ is 65.2. Using 65.2 in the same equation, but this time solving for $\frac{\lambda_p}{\lambda_m}$ gave a scale of 9.498, which was entirely satisfactory.

5. Preparation of the Model

As mentioned previously, other investigators have remarked that suitable thin-wall tubing was not readily available. In the present investigation it was found that suitable tubing was available -- although a smaller diameter would have been more desirable -- but that the real difficulty lay in bending the tube into the required form for the model. From the beginning it was realized that bending without crimping would not be easy, and to allow for initial failures enough yellow brass tubing was ordered to make four models. This material was chosen for its ease of bending. (As described in paragraph 6, however, this material proved to have inadequate yield strength in the softened condition.) As it turned out, all the material was actually used. To support the tube during the bending operations it was filled with a low-melting alloy, "Fusible Alloy No. 158," a product of the American Smelting and Refining Company. It was thought that, thus supported, the tube could be bent with a ram type bender locally available. However, this proved far from true, for on almost the first touch of the ram the tube crimped. Eventually the three remaining lengths of tubing were taken to the San Francisco Naval Shipyard. The tubing was flame-annealed, and after two unsuccessful attempts one satisfactory model was obtained from a large compression-bending machine and ball-head mandrel but without the use of any filler. In addition to the unavoidable thinning of the walls on the outsides of the bends the resulting model actually had the following defects:

The portion between the two bends was distorted for about four inches as shown roughly in Fig. 5.

Instead of being exactly 90 degrees, one bend was 89 degrees and the other, 86 degrees.

Once the tube was bent, the mirror-mounting barrel was slipped over one end and the two end-plates silver-soldered in place.

6. Procedure

(a) Preparation of the Apparatus

In addition to simple rearrangement of the various components of Rauch's apparatus, it was found necessary to raise the compound feed device nine inches to accommodate the model to be used. This was accomplished by constructing a sturdy, table-like foundation and welding it in place on top of the existing foundation. (Fig. 3). The compound feed was then aligned and bolted in place.

The assemblies were roughly aligned and clamped in place. To adjust the deformation assembly relative to the anchor point in such a way that exactly the right X, Y, and Z dimensions were provided, the two end-plates were temporarily installed (subsequently to be removed and silver-soldered to the tube as described in paragraph 5). An eight-inch length of one-eighth inch diameter steel rod, threaded at one end and having a thin slot in the other end, was screwed into the hole provided for it in the center of the end-plate mounted on the anchor plate. A small, precision plumb bob was suspended from the end of the rod, supported by the friction of its thread in the slot. When adjusted so that it just touched the floor, the point of contact was marked and the height measured. Rod and bob were then removed and installed in the other end-plate (mounted on the deformation assembly), which could then be positioned to its correct height. To achieve the correct horizontal position, the deformation assembly was moved until the plumb bob touched a point marked on the floor by measurements from the point previously marked for the anchor.

(b) Installation of the Model

Upon proceeding to bolt the model in place by means of its end-

plates, it was found that the holes in the end-plate at the anchor end did not quite line up with the holes in the anchor plate. The end-plate was, therefore, removed, the end-plate bolted in place, and then the tube was again silver-soldered to its plate.

During the subsequent initial testing of the deformation assembly it was noticed that both ends of the tube had become badly crimped next to the supports -- to such an extent that any results would probably be invalidated. Rather than to construct a whole new model, it was decided simply to cut off the deformed parts (about three-quarters of an inch), reinstall the model, and make the necessary slight changes in the initial problem dimensions. This was done without removing either end-plate from the apparatus.

Three sets of readings were then taken for each of the three deformations ratios (0.25, 0.50, 0.75). Since it was now known that the end-plates were not installed accurately enough to permit simple reversal of the tube without putting it under considerable initial stress, the model was to be removed from its end-plates (by melting the solder), reversed, and re-soldered. Unfortunately during this process a hole was burned through the tube at one end. This required cutting off the damaged portion (about one-half inch) and making another (final) alteration to the initial problem dimensions. The resulting dimensions are those shown in Fig. 4.

(c) Measurement of Deflections

As mentioned earlier, Rauch's system was used for measuring both translation and rotation. With the cantilever length, L , of 10.25 inches chosen, readings were being taken just short of the beginning of

the bend closest to the anchor point. Three deformation ratios were used -- 0.25, 0.50, and 0.75 -- in order to determine the effect any variation of this ratio might have on the results when a tubing model was used. Half the total deformation was applied in the negative direction, and readings of y'' , z'' , ξ , η , and ζ taken. Then the same deformations were applied in the positive direction and the same readings taken. This sequence was followed at least three times for each deformation ratio. First results were not encouraging; readings obtained with the same deformations were not repetitive. But as the procedure of shifting from negative to positive deformations was repeated again and again correlation improved markedly until finally it was deemed satisfactory. It is believed by the writer that the improvement was the result of working out stresses locked in the tube by the heat applied at its ends during soldering.

The deformations thus measured were sufficient to lead to all reactions except the axial force P_x . To obtain the deformations necessary to calculate this force and to check the other reactions the tube was reversed and another set of readings taken using the same deformation ratios.

Three points of technique are considered worthy of mention. First, in the measurement of translation it was found essential to back off one knife edge a set, exact amount before taking a reading with the other knife edge. This procedure ensures that all readings are taken at the same point on each knife edge and removes the effects of any misalignment of the edges.

A second point is that in moving the translation-measuring device great care must be taken not to drive the knife edges hard against

the "riser-sustainer" barrel, for, unlike a rod model, the heavier tube will not give way, and the alignment of the knife edge will be changed enough to remove the possibility of correlation of subsequent readings with previous ones.

Finally, it was found possible in measuring rotations to record the amount of motion of the reflected points of light without resorting to marking with a pencil their initial and final positions as Rauch did. The mirrors used were good enough so that the points of light were still visible under a dim exterior light, and thus their positions could be read and recorded directly. It is felt that such a procedure leads to more accuracy in this critical measurement than does trying to mark the points in complete darkness.

Following tests with the tube model, another model of the same prototype was constructed, this one of one-eighth inch diameter steel rod using the same linear scale and the same L as was used with the tube. After the slight rearrangement of the main instrument assembly necessary to accommodate the model, it was installed and a test made with a deformation ratio of unity. The purposes of this test were twofold: to demonstrate directly the difference in indicated reactions caused by the flexibility factor and to check on the investigator's technique (since the approximate degree of accuracy obtainable with this type of model was known from Rauch's work).

(d) Analytical Calculations

A complete solution to the two problems -- $k = 1$ for the rod and $k = 3.357$ for the tube -- was obtained by Professor John E. Brock, using his matrix method [3 and 4] on a digital computer. In both in-

stances the solutions achieved were for point O, and reactions at point A were found by statics.

7. Solutions

The deflections for all runs for each deformation ratio were averaged. Rotations were corrected using Rauch's calibration curves for the optical systems. Table 1 lists equations then used to solve for reactions. The equations for the rod were taken directly from Rauch's thesis. Those for the tube are also based on Rauch's, but because deflection due to shear was a factor to be considered with the tube, the equations were modified as shown. Appendix I gives the method of arriving at these modifications. Calculations of the various multiplying factors are shown in Table 2. Table 3 combines a summary of deflections observed and calculations of reactions for the two models in both the normal and reversed positions. Tables 4 and 5 summarize results for the tube and the rod respectively. Values of R_y are averages of those obtained in the normal and the reversed positions. Each table includes the corresponding values of all reactions as obtained by Professor Brock's digital computer solution. Using the computed values as correct, per cent errors for all experimental values have been computed and listed in the two tables.

3. Results

It can be seen from a perusal of Table 3 that accuracy of the results of tests using the rod are comparable to those obtained by Rauch using a wire. Although the per cent errors of the two smaller forces, F_1 and F_2 , are quite high, it is noted that the resultant force, F_R , is well within reason.

In contrast with the rod, examination of the results for the tube as summarized in Table 4 shows them to be far from satisfactory. The errors vary widely among the various moments for each deflection ratio. In general, any one reaction shows an increasing per cent error with increasing deflection ratio. One other broad generalization which may be made as these errors in moments are considerably smaller than those in forces.

Other than the above generalizations no particular pattern could be observed which would lead to discovery of some mistake in the theory used or to some error aspect of technique. It had been expected that dimensional errors in the total model load would cause minor errors in results, but certainly not errors as large as those actually encountered. Actually the dimensional errors of the tube were considered to be of the same order as those in the rod caused by its drooping under its own weight. In trying to understand the cause of the large reaction errors more consideration was given to the possibility of deformation of the supporting structure, though it was not until forces were measured at the supports were far removed from the tube that this was apparent. When a few tests made by simply putting the rod on the supports and the apparatus were dismantled, it was found that deformation of the supporting structure of the apparatus would be most likely to lead to indicated reactions erring on

the small side, whereas the forces indicated in Table 3 are all too large.

It is believed, rather, that the main cause of the large errors lay in stressing the tube beyond its elastic limit. It will be recalled that silver-soldering was used to secure the two end-plates to the tube and that this process, which requires a temperature of 1000°F, was repeated several times. As the temperature along the tube during soldering decreased rapidly with distance from the point of application of the heat, the tube was subject to various degrees of tempering. Recall also that it had been flame-annealed before bending. The yield point for yellow brass is approximately 11,000 psi [11] in the softened condition as against 57,000 psi in the hardened condition. An investigation (Appendix II) was made of the stresses at the anchored end of the tube in its normal position and under the smallest deformation ratio used, 0.25. The analysis showed that at one point the stress was 17,417 psi, which is well above the yield strength for the softened condition. For the higher deformation ratios used this figure would have been 34,834 and 52,251 psi.

If the tube had, in fact, been stressed beyond its yield strength it seemed logical that it should not return exactly to its no-load position in space when the load was removed -- that this position should change with each loading. Accordingly a check was made of the constancy of the position of the point of measurement P in the unloaded, or neutral, position. Because of the manner in which deformations had been applied -- that is, first in the negative direction and then in the positive and back again -- there had been no evident necessity for

taking readings in the neutral position. Therefore it was necessary to obtain an approximation to this position by assuming that it lay half way between those for the positive and the negative deformations. The readings y_1' and y_2' , z_1' and z_2' etc. for each deformation ratio were averaged and the results assumed to be the position of P for each ratio. A very definite "drifting" of the point was indicated, bearing out the theory that inelastic behavior had occurred.

It is unfortunate that time did not permit re-running of the experiment with a new model of different material.

9. Conclusions

Despite the poor results obtained in the tests described herein, the investigator feels that the basic method of accounting for moderate bend flexibility factors by the use of tubing for models is not without promise. Let us examine the difficulties and failings encountered and see how they might be overcome. First there is the model itself. It must be small enough in cross-section to be handled with a fair amount of ease, and it must be small enough in overall size to be accommodated within the existing apparatus. These two requirements combine to demand a thin-walled tube. Because of the difficulties attendant on bending such a tube, the work should be done by experienced personnel in a well-equipped shop; dies and mandrels must be of the exact size. Even under these conditions there will inevitably be a certain amount of thinning of the tube wall at the outside of bends and a corresponding thickening at the inside, but this should be similar to conditions in prototype pipe which has been bent. It should be noted too that there is a minimum limit to the radius which can be produced in any particular tube and thus to bends which can be simulated. These limits are tabulated in such books as reference 12, which also contains a great deal of other information on bending of pipe and tube. Welding elbows, for example, can probably not be simulated with tubing. In any case, the material selected for the model should have high ductility for ease of bending and a low modulus of elasticity to minimize the actual reactions on the apparatus. It should also be heat treatable in order to obtain uniform properties after installation of the end-plates. Since the end-plates should not have to be removed if properly installed initially, the mirror-mounting barrel should be made in

two tubes to permit clamping it from one end of the model to the other when the model is reversed. The author suggests stainless steel as a suitable material because, although the modulus of elasticity is high, it can easily be bent, can be heat-treated, and is available in an extremely wide variety of sizes with thin walls.

Supposing that a satisfactory model were obtained and mounted, there would still remain the possibility of stressing it beyond its elastic limit by applying deformations which were too large. (In the case investigated the maximum stress for deformation ratio 0.75 was extremely close to the yield point even for the hardened condition of the material.) No prior calculation of these stresses would be possible, of course, because the forces and moments causing them are the very reactions being sought by the tests. To reduce the danger of overstressing the model, then, it would seem advisable to use an extremely low initial deformation ratio, solve for the reactions, and, from them, for the maximum stress. The largest possible deformation ratio usable without exceeding the elastic limit could then be calculated by multiplying the initial deformation ratio used by the ratio of yield strength to the maximum stress determined in the initial test (employing some suitable safety factor). It might well be, however, that the maximum allowable deformation ratio would be so low that the actual deformations would be too small to be determined with much accuracy, but it is not believed that such would be true in most cases.

In summary the investigator feels that the method of accounting for bend flexibility factors by the use of tubing models is one which should not be overlooked by persons interested in analyzing piping flexibility.

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APPENDIX I

MODIFICATION OF EQUATIONS RELATING DEFLECTIONS TO REACTIONS

Consider the portion of the tube between the point of measurement P and the anchor as a cantilever beam under the influence only of a shear force F. The slope of the resulting deflection curve (10) is

$$\theta = \frac{dy}{dx} = \frac{Ss_{\max}}{G}$$

For vertical loading the maximum shear stress occurs in the horizontal plane of the neutral axis and is given to a good approximation by the formula

$$Ss_{\max} = \frac{FQ}{Ib}$$

In the case of a tube, b is 2t. Also

$$Q = 2 \int_0^{\pi/2} (r \sin \gamma) t r d\gamma = 2 r^2 t$$

where γ is the angle between any radius line and the horizontal, measured counterclockwise. Finally

$$G = \frac{E}{2(1+\mu)}$$

Substituting into the slope equation gives

$$\theta = \frac{(F)(2r^2t)}{\frac{E}{2(1+\mu)}(I)(2t)} = \frac{2(1+\mu)r^2F}{EI}$$

Letting $g = 2(1+\mu)r^2$, we have

$$\theta = \frac{gF}{EI}$$

Integrating the slope from $x = 0$ to $x = L$, we arrive at the deflection at P due to shear,

$$\delta_s = \frac{gFL}{EI}$$

In the experiments employing the tubular model, then, deflections read are the results of both bending and shear. Since Bauch's equations (Table 1) consider bending only, the deflections read must be decreased by that part of those deflections due to shear. That is,

$$D_n = D'_n - D_{n,s}, \text{ where}$$

D_n is the deflection to be used in the equation;

D'_n is the corresponding deflection observed in the test;

$D_{n,s}$ is the corresponding part of the observed deflection which is due to shear.

Not let δ_2 and δ_3 be shear translations in the y'' and z'' directions and θ_4 , θ_5 , and θ_6 be rotations about the x'' , y'' and z'' axes, respectively, due to shear. Then for Equation 5 (Table 1):

$$\begin{aligned} F_2 &= \frac{GEI}{L^2} \left[\frac{2}{L} (D_2 - \delta_2) + (D_6 - \delta_6) \right] \\ &= \frac{GEI}{L^2} \left[\frac{2}{L} \left(D_2 - \frac{L}{EI} g_{F_2} \right) + \left(D_6 + \frac{1}{EI} g_{F_2} \right) \right] \\ &= \frac{6EI}{L^2 + 3g} \left(\frac{2}{L} D_2 - D_6 \right) \end{aligned} \quad (5')$$

Similarly for Equation 6:

$$F_3 = \frac{GEI}{L^2 + 6g} \left(\frac{2}{L} D_3 - D_5 \right) \quad (6')$$

Shear does not affect F_4 ; therefore Equation 7 is unaltered.

For Equation 8:

$$\begin{aligned} F_5 &= \frac{GEI}{L^2} \left[\frac{L}{3} (D_5 - \theta_5) - (D_3 - \delta_3) \right] \\ &= \frac{GEI}{L^2} \left[\frac{L}{3} \left(D_5 - \frac{1}{EI} g_{F_5} \right) - \left(D_3 - \frac{L}{EI} g_{F_3} \right) \right] \end{aligned}$$

$$= -\frac{6EI}{L^2} \left(\frac{L}{3} D_5 - D_5 \right) - 4 \frac{L}{3} F_3 \quad (8')$$

Similarly it can be shown that Equation 9 becomes:

$$F_6 = -\frac{6EI}{L^2} \left(\frac{L}{3} D_6 - D_2 \right) - 4 \frac{L}{3} F_2 \quad (9')$$

It is assumed that shear deflection in the prototype is negligible because of its very large ratio of length to diameter. All customary methods of analysis make this assumption, although the effect can be incorporated in some analytical treatments.

APPENDIX II

ANALYSIS OF STRESSES AT POINT A

Consider the conditions at the anchored end of the tube (Point A) with the deformation ratio of 0.25 applied. By applying the proper scale factors to the computer solution we find that the forces and moments acting on the tube are as follows:

$$\begin{array}{ll} -F_x = -11.3 \text{ lbs.} & -M_x = -108.3 \text{ in. lbs.} \\ -F_y = -52.3 \text{ lbs.} & -M_y = 211.5 \text{ in. lbs.} \\ -F_z = -17.31 \text{ lbs.} & -M_z = 382.0 \text{ in. lbs.} \end{array}$$

we now find the resultant of M_y and M_z , which is

$$M_R = 461 \text{ in. lbs.}$$

acting in the $y' - z'$ plane as shown in Fig. 6 (b).

Next we find the resultant of F_y and F_z , which is

$$F_R = 53.6 \text{ lbs.,}$$

acting in the $y' - z'$ plane as shown in Fig. 6 (a).

It can now be seen that the axes of F_R and M_R are fortuitously close to orthogonality. For the sake of simplicity they will be considered exactly orthogonal and designated x'' and z'' as shown in Fig. 6 (c). Let us now focus our attention at the point of intersection of the negative y'' axis with the outside surface of the tube. At this point the shear stress due to F_R is zero, and the normal stress due to M_R is

$$S_n = \frac{M_R r_o}{I} = 17350 \text{ psi.}$$

The shear stress due to M_x at the outside of the tube is everywhere

$$S_s = \frac{M_x r_o}{J} = 2120 \text{ psi.}$$

which, the normal stress for σ_{xx} is everywhere

$$\sigma_{xx} = \frac{F}{A} = 100 \text{ psi, where } A \text{ is the cross-section area of the tube.}$$

The stresses calculated above are used to produce a Mohr circle (Fig. 3(d)). This shows a maximum normal (principal) stress of 17417 psi. Since this is a linear system, the stresses at $\frac{\Delta_m}{\Delta_p} = 0.50$ and 0.75 should be two and three times this amount, respectively.

The equations used herein are valid only for elastic behavior. Since the yield point of the brass used in the tubing model was only 11,000 psi, it is evident that this point was exceeded, and thus the quantitative results of this analysis serve only to show that inelastic action must have occurred.

APPENDIX III

THE USE OF COIL SPRINGS TO SIMULATE PIPING FLEXIBILITY

It is desired to know whether coil springs could replace the bends of a solid rod model to simulate the bend flexibility behavior of a pipe. In piping the ratio of torsional rigidity (referred to herein as τ) to flexural rigidity in a bend (β) is:

$$\frac{\tau}{\beta} = \frac{k}{(1+\mu)}$$

Flexural rigidity of a spring with wire of circular cross-section is [8]:

$$\beta = \frac{1}{n} \frac{E I G}{\pi \bar{r} (2G+E)}, \text{ where:}$$

l_0 is the free length of wire,

n is the total number of turns of wire,

$\bar{r}$ is the mean radius of the coil, and

I is the moment of inertia of the cross-section of the wire.

The corresponding torsional rigidity of such a spring is [9]:

$$\tau = \frac{1}{\frac{\cos^2 \alpha}{EI} + \frac{\sin^2 \alpha}{GJ}}$$

where α is the angle of inclination of the coils to a circular section of the cylinder on which the coil lies. E , I and J refer to the wire composing the spring.

To simplify the form of the ratio of the two rigidities, β and τ , we first reduce each to the terms of E , I , α , and μ as follows:

$$\frac{l_0}{n \pi r} = 2 \frac{\frac{l_0}{2n}}{\pi r} = 2 \sec \alpha$$

$$G = \frac{E}{2(1+\mu)}$$

$$J = 2 I$$

$$\theta = \frac{2 \times 2 \sec \alpha EI \frac{E}{2(1+\mu)}}{2 \frac{E}{2(1+\mu)} + E} = \frac{2 EI \sec \alpha}{2 + \mu}$$

$$T = \frac{1}{\frac{\cos^2 \alpha}{EI} + \frac{\sin^2 \alpha}{\frac{E}{2(1+\mu)} (2 I)}} = \frac{EI}{\cos^2 \alpha + (1+\mu) \sin^2 \alpha}$$

$$\frac{T}{\theta} = \frac{EI}{\cos^2 \alpha + (1+\mu) \sin^2 \alpha} \times \frac{2EI \sec \alpha}{2 + \mu} = \frac{(2+\mu) \cos \alpha}{2 [\cos^2 \alpha + (1+\mu) \sin^2 \alpha]}$$

Equating this expression to the desired ratio and cross-multiplying we have

$$k = \frac{(1+\mu) (2+\mu) \cos \alpha}{2 \cos^2 \alpha + (1+\mu) \sin^2 \alpha} \quad (10)$$

For the case of a steel spring $\mu = 0.3$, and

$$k = \frac{2.99 \cos \alpha}{2 (\cos^2 \alpha + 1.3 \sin^2 \alpha)}$$

Differentiating k with respect to α ,

$$\frac{dk}{d\alpha} = \frac{-7.92 \sin^3 \alpha - 1.794 \sin \alpha \cos^2 \alpha}{4 (\cos^2 \alpha + 1.3 \sin^2 \alpha)^2}$$

It can be seen that within the conceivable range of α 's for a coil spring (0 to $\frac{\pi}{2}$) k decreases with increasing α , so that the maximum value of k which can be realized would be with $\alpha = 0^\circ$. With such a spring, using equation (10),

$$k = \frac{2.99}{2} = 1.50.$$

TABLE 1

EQUATIONS RELATING OBSERVED DEFLECTIONS TO REACTIONS

REACTION	EQUATION NUMBER	RAUCH'S EQUATION ¹	EQUATION NUMBER	MODIFIED EQUATION ¹
F_2	5	$\frac{6EI}{L^2} \left(\frac{2}{L} D_2 \neq D_6 \right)$	5'	$\frac{6EI}{L^2 \neq 6\epsilon} \left(\frac{2}{L} D_2 \neq D_6 \right)$
F_3	6	$\frac{6EI}{L^2} \left(\frac{2}{L} D_3 - D_5 \right)$	6'	$\frac{6EI}{L^2 \neq 6\epsilon} \left(\frac{2}{L} D_3 - D_5 \right)$
F_4	7	$\frac{JG}{L} D_4$	7	$\frac{JG}{L} D_4$
F_5	8	$-\frac{6EI}{L^2} \left(\frac{L}{3} D_5 - D_3 \right)$	8'	$-\frac{6EI}{L^2} \left(\frac{L}{3} D_5 - D_3 \right) \neq 4 \frac{L}{\epsilon} F_3$
F_6	9	$-\frac{6EI}{L^2} \left(\frac{L}{3} D_6 \neq D_2 \right)$	9'	$-\frac{6EI}{L^2} \left(\frac{L}{3} D_6 \neq D_2 \right) - 4 \frac{L}{\epsilon} F_2$

¹ D_4 , D_5 , and $D_6 = (\xi_2 - \xi_1)$, $(\eta_2 - \eta_1)$ or $(\zeta_2 - \zeta_1)$, respectively

times correction factor from optical system

calibration curves (8) times 10^{-3} .

Calculations

(a) For Δ_m term

$$\frac{2}{L} = \frac{2}{10.25} = 0.1951$$

$$\frac{1}{L} = \frac{10.25}{2} = 0.0976$$

$$\frac{6EI}{L^3} = \frac{6 \times 15.7 \times 10^6 \times 0.06}{10.25^3} = 1.412 \times 10^4$$

$$\frac{JG}{L} = \frac{2 \times 0.1628 \times 10^6}{10.25} = 1.274 \times 10^4$$

$$\delta^* = 2 \times (1 + \mu) r^2 = 2 \times (1 + 0.3) 1.228^2 = 4.0$$

$$\frac{6EI}{L^3 + \delta^*} = \frac{6 \times 15.7 \times 10^6 \times 0.06}{10.25^3 + 4.0 \times 6} = 1.173 \times 10^4$$

$$\frac{4}{L} \delta = \frac{4}{10.25} \omega = 1.561$$

$$\frac{F_p}{F_m} = \frac{E_p I_p \lambda_p^2 \Delta_m}{E_m I_m \lambda_p^2 \Delta_p} = \frac{27.9 \times 10^6 \times 5621 \times 1 \times \Delta_m}{15.7 \times 10^6 \times 116 \times 1.475 \Delta_p}$$

$$= 71.93 \frac{\Delta_m}{\Delta_p}$$

$$\frac{M_p}{M_m} = \frac{F_p \lambda_p}{F_m \lambda_m} = 71.93 \frac{\Delta_m}{\Delta_p} \times 1.115 = 80.29 \frac{\Delta_m}{\Delta_p}$$

For $\frac{\Delta_m}{\Delta_p} = 0.25$: $\frac{F_p}{F_m} = 18.0$, $\frac{M_p}{M_m} = 20.3$

For $\frac{\Delta_m}{\Delta_p} = 0.5$: $\frac{F_p}{F_m} = 35.96$, $\frac{M_p}{M_m} = 40.15$

For $\frac{\Delta_m}{\Delta_p} = 0.75$: $\frac{F_p}{F_m} = 53.94$, $\frac{M_p}{M_m} = 60.21$

Table 2. Calculation of Multipliers

(b) For thread

$$\frac{SEE}{L} = 20.532$$

$$\frac{JG}{L} = 26.892$$

$$\frac{F_p}{F_m} = 50,900$$

$$\frac{M_p}{M_m} = 483,500$$

Table 2 (cont) Continued of M_{eff} data.

Operation	Re- sult	Sign	$\Delta \rightarrow 1$ $\Delta/\Delta_F \rightarrow .25$	2 .50	3 .75	4 .25	5 .50	6 .75	7 1.0	8 1.0
$(2/L) D_2$		-	.01012	.02129	.03415	.01189	.02345	.03869	.04706	.04646
$-D_2$		-	.00740	.01530	.02360	.00770	.01610	.02470	.03420	.03480
Alg. Sum.		-	.00272	.00599	.01058	.00399	.00735	.01399	.01286	.01166
$X(6EI/(L^2+6E)) F_{2m}$		-	33.48	73.6	130.0	49.1	90.4	172.0	0.264	0.2395
$X F_P/F_m$	$F_{y'p}$	-	8630	13600	12460	14130	13010	16480	18450	12210
$2/L D_3$		-	.00704	.01526	.02314	.00784	.01594	.02770	.03858	.03746
$-D_3$		-	.00510	.01040	.01530	.00670	.01170	.01860	.03300	.03200
Alg. Sum.		-	.00194	.00486	.00724	.00214	.00424	.00910	.00558	.00546
$X(6EI/(L^2+6E)) F_{3m}$		-	23.85	49.75	39.0	263	52.1	112	0.1147	0.1122
$X F_P/F_m$	$F_{2'p}$	-	6860	8600	8530	7570	7500	10730	5840	5710
D_4		-	.00570	.0113	.0175	.00800	.0106	.0165	.0208	.0186
$(JG/L) D_4$	F_{4m}	-	107.0	212.0	329.0	33.8	138.8	309.2	.5500	.5010
$X M_P/M_m$	$M_{x'p}$	-	292,000	289,400	299,000	256,200	271,400	281,500	286,000	242,000
$1/3 D_5$		-	.0174	.0355	.0643	.0195	.0400	.0635	.1128	.1093
$-D_5$		-	.0361	.0725	.1196	.0402	.0817	.1420	.1977	.1920
Alg. Sum.		-	.0187	.0370	.0643	.0207	.0417	.0785	.0849	.0827
$X(6EI/L^2)$		-	272.0	527.5	935.0	301.0	606.0	1141	1.745	1.700
$(L/E) F_{3m}$		-	30.0	59.0	134.2	39.7	78.6	169	0	0
Alg. Sum.	F_{6m}	-	236.0	479.5	800.8	261.3	527.4	972	1.745	1.700
$X M_P/M_m$	$M_{y'p}$	-	645,000	654,000	720,000	714,000	721,000	885,000	844,000	823,000
$1/3 D_6$		-	.0253	.0523	.0806	.0263	.0550	.0844	.1169	.1190
$-D_6$		-	.0519	.1091	.1752	.0599	.1202	.1933	.2412	.2381
Alg. Sum.		-	.0266	.0568	.0946	.0336	.0652	.1139	.1243	.1211
$X(6EI/L^2)$		-	386.6	826	1380	489	949	1655	2.455	2.490
$(L/E) X F_{2m}$		-	50.4	111	196	74	135	259	0	0
Alg. Sum.	F_{6m}	-	336.1	715	1184	415	814	1396	2.555	2.490
$X M_P/M_m$	$M_{z'p}$	-	919,000	978,000	1,078,000	1,134,000	1,112,000	1,270,000	1,236,000	1,203,000

- NOTES: (1) This table constitutes a summary of the observed deflections and the calculations leading therefrom to the reactions at Point A.
(2) The calculated reactions are those exerted by the pipe or tube on the structure.
(3) Columns 1 through 6 are for tests with the tubing model; columns 7 and 8, for the rod model.
(4) Odd-numbered columns are for the model in the normal position; even-numbered, for the reversed position.
(5) For the rod model, g is considered zero.

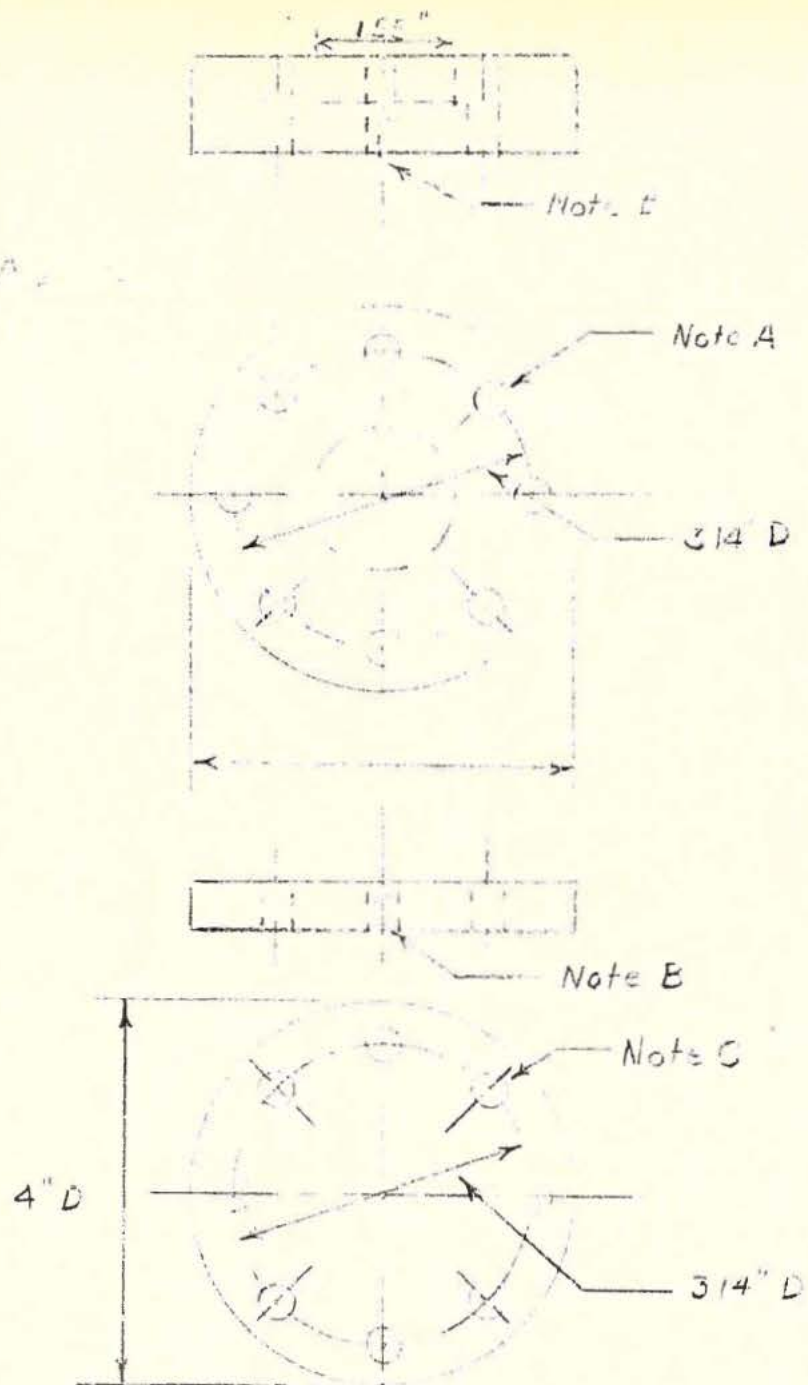
Table 3. Deflections Observed and Calculations made Therefrom.

Re-Action Sign	Computer Value	$\frac{\Delta m}{\Delta p} = .25$	% Error	$\frac{\Delta m}{\Delta p} = .5$	% Error	$\frac{\Delta m}{\Delta p} = .75$	% Error
Fxo -	4,761	7,570	59.0	7,500	57.5	10,730	124
Fyo -	9,363	11,380	26.9	11,205	26.1	12,735	36.0
Fzo +	4,762	8,260	44.1	8,600	30.7	8,530	79.1
Fo -	11,533	15,640	35.7	15,370	42.1	18,650	61.9
Mxo -	1,065,205	1,134,000	5.5	1,112,000	4.4	1,270,000	19.2
Myo +	573,500	714,000	23.5	721,000	24.7	885,000	53.2
Mzo +	297,129	256,200	-13.8	271,400	-8.8	281,500	-5.4
Mo -	1,242,063	1,331,000	10.8	1,354,000	8.6	1,593,000	27.7
Fx _A +	4,761	7,570	59.0	7,500	57.5	10,730	124
Fy _A +	9,363	11,380	26.9	11,205	26.1	12,735	36.0
Fz _A -	4,762	8,260	44.1	8,600	30.7	8,530	79.1
F _A -	11,533	15,640	35.7	15,370	42.1	18,650	61.9
My _A +	296,000	292,000	-1.3	289,400	-2.4	299,000	1.0
My _A -	577,000	645,000	11.8	654,000	13.3	729,000	26.4
Mz _A -	1,071,000	919,000	-14.1	978,000	-3.7	1,078,000	0.7
M _A -	1,250,000	1,155,000	-7.6	1,210,000	-3.2	1,335,000	6.8

Table 4. Summary of Results using Tubing Model.

Readings	Sign	Computer Value	Test Value	Per cent Error
P ₂₀	-	7,382	5,710	-22.7
P ₃₀	-	15,378	12,830	-16.1
P ₄₀	+	7,380	5,840	-20.9
P ₅₀	-	10,968	15,150	+10.7
P ₆₀	-	1,239,370	1,243,000	+ 2.9
P ₇₀	+	903,533	881,500	- 2.5
P ₈₀	+	240,000	242,000	+ 0.9
P ₉₀	-	1,550,633	1,542,000	- 0.7
P ₁₀₀	+	7,382	5,710	-22.7
P ₁₁₀	+	15,378	12,830	-16.1
P ₁₂₀	-	7,380	5,840	-20.9
P ₁₃₀	-	10,968	15,150	+10.7
P ₁₄₀	+	243,000	246,000	+ 1.4
P ₁₅₀	-	903,000	883,500	- 2.0
P ₁₆₀	-	1,341,000	1,257,000	- 6.4
P ₁₇₀	-	1,548,000	1,512,000	- 2.0

Table 3. Summary of Results using Rod Model.



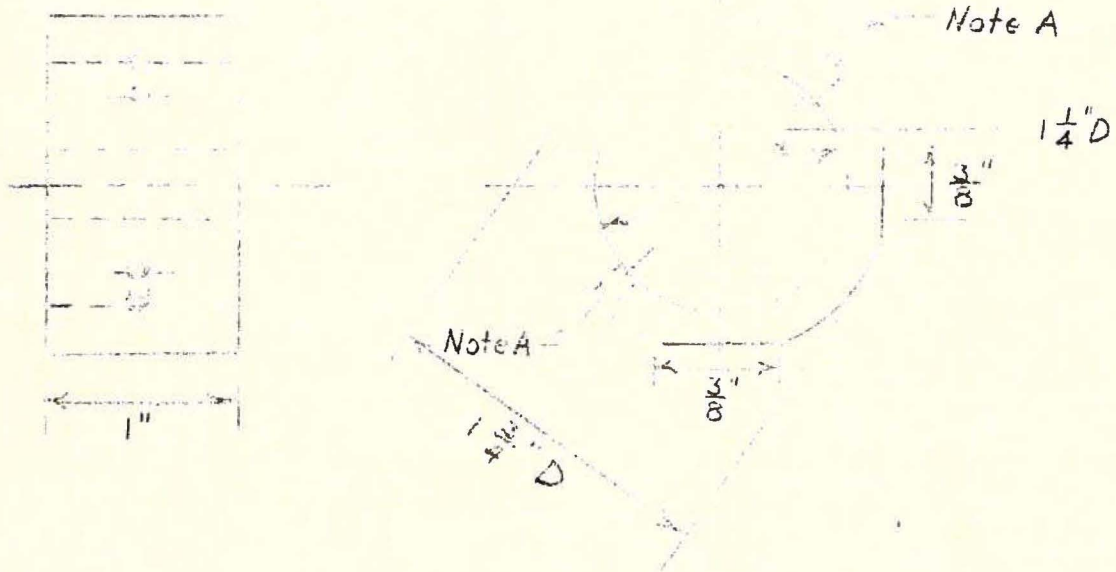
NOTES: A - Drill and tap all holes on 3.14" D for 5/16" cap screw.

B - Drill and tap for 1/8" NC thread.

C - Drill only for 5/16" cap screw.

D - Material: mild steel.

Figure 1. Milling head adapter and end-plate.



NOTES: A - Drilled and tapped for 1/8 inch set screw.

B - Material is brass with outside surfaces silver-plated.

C - Front-silvered mirrors mounted on each 3/8 inch flat.

Figure 2. Mirror-Mounting Barrel

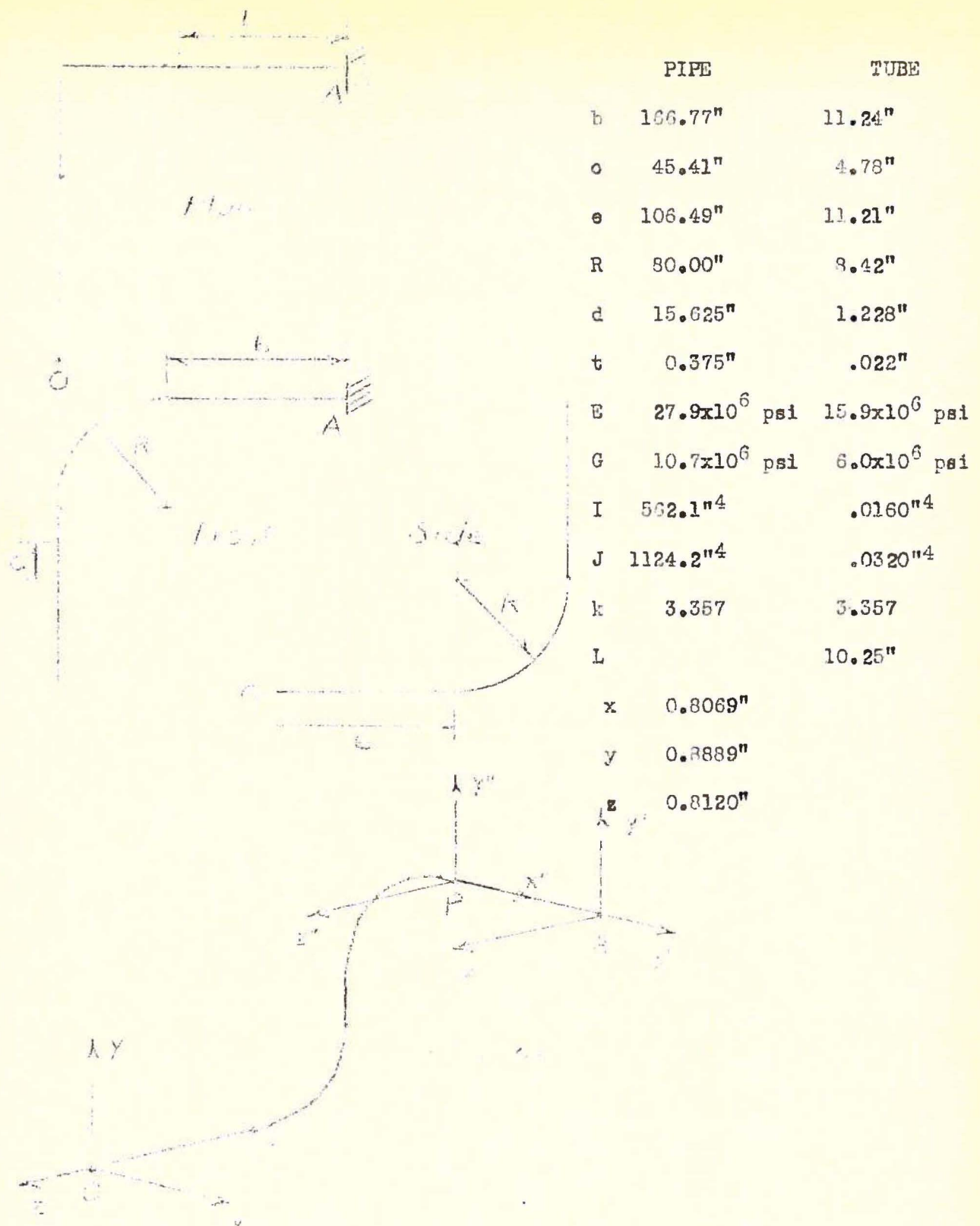


Figure 4. Dimensions and Required Properties of Prototype System and Tubing Model.

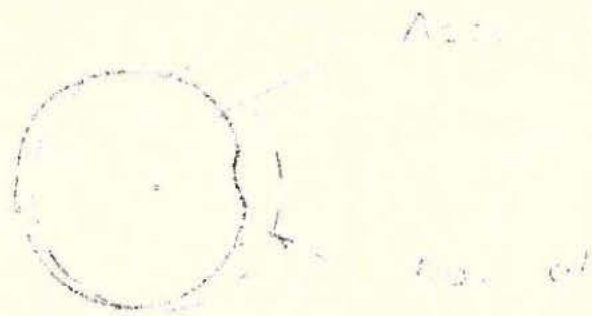


Figure 3. Rough Profile of Distorted Cross-Section
Obtained by use of Modelling Clay

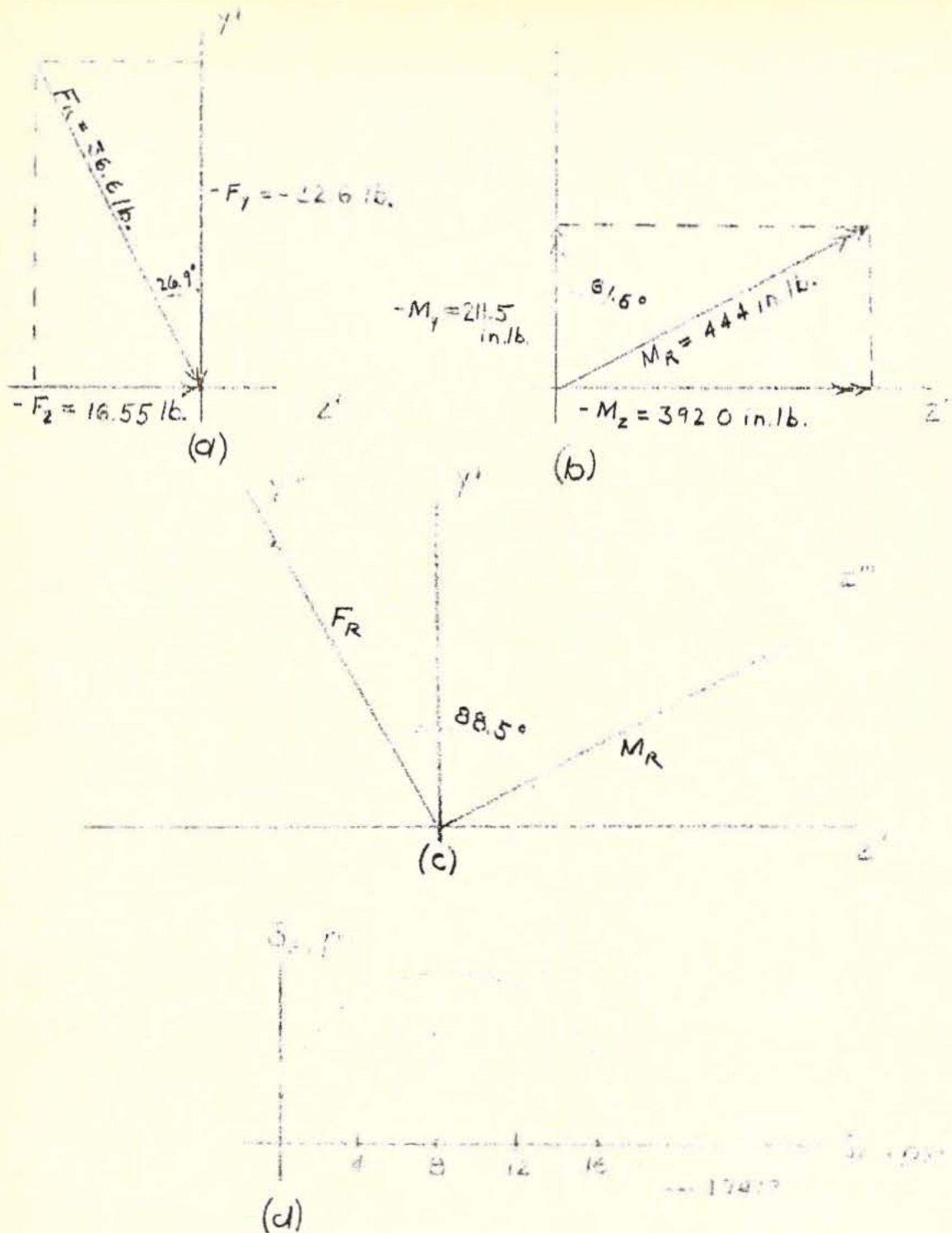


Figure 3. Graphical Portrayal of Forces, Moments and Stresses at Point A.

Westinghouse

ELECTRIC CORPORATION



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Lt. Peter H. Freeman, U.S.N.
Box 1465, U.S. Naval Post
Graduate School
Monterey, California

December 18, 1957

Dear Lt. Freeman:

Subject:- Model of Corrugated Pipe.

This is in reply to your letter of December 7 on the above subject. During the period from 1938 to 1945 we made several pipe line models for checking flexibility. These lines contained corrugated tangents. The relative torsional and lateral flexibility of these corrugated parts was simulated by use of slotted tubes as mentioned in the "Power" article referred to in your letter.

Attached is a drawing of one of these slotted tubes with the expected values of I and J given at the bottom. I am not sure that the test values were exactly the same as the expected values, however, I feel quite sure that the relation of these two numbers was closely approximated, either with the slots as shown or with a slight increase in depth of the same.

We were never able to work out an analytical method for designing the slotted tubes, but instead used a "cut-and-try" approach to the problem.

After obtaining a slotted tube with the desired relative flexibility, the remainder of the model of the pipe system is chosen in a manner to be compatible with the slotted tube which simulates the corrugated portion of the pipe system.

Most of our piping systems have four or less anchor points. Such piping system can be readily analyzed by use of computing machines. Therefore, we have not used model testing for a number of years and have, therefore, no incentive for further developments in the type of problem in which you are interested.

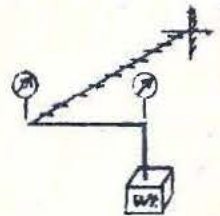
Very truly yours,

J. D. Conrad
J. D. Conrad, Sr.
Asst. Mgr., Large Turb. Engrg.

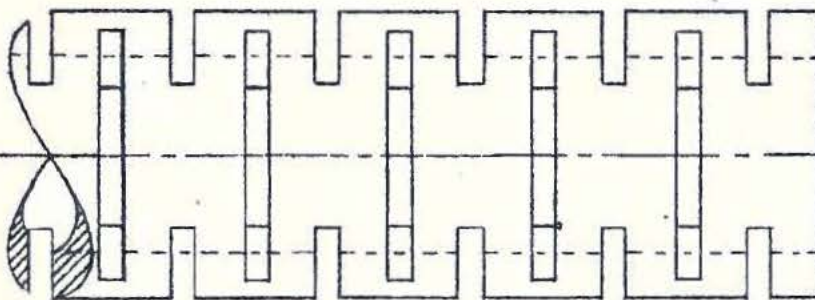
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YOU CAN BE **SURE**... IF IT'S Westinghouse

ENGINEERING DEPT. CALCULATION REPORT.



Test arrangement

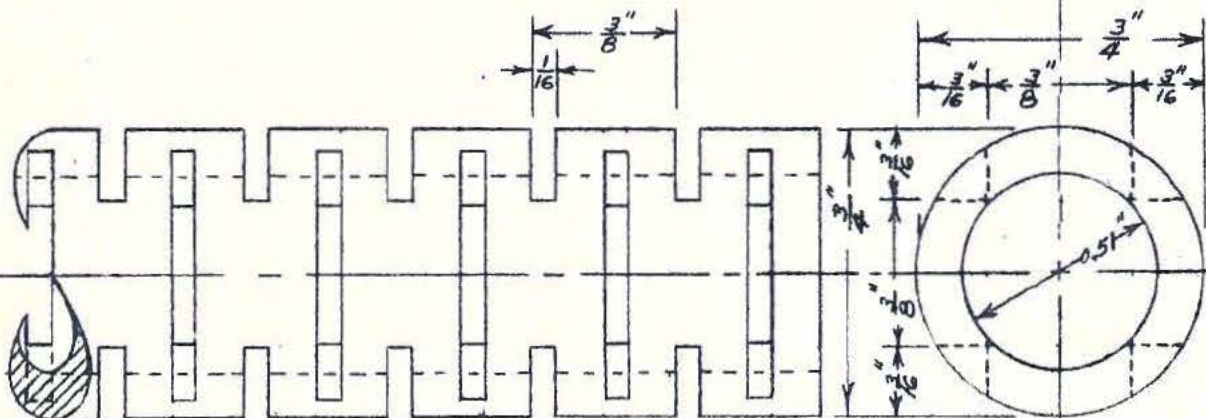


Top view

$\frac{3}{16}$ "



$\frac{1}{16}$ " R.
Suggested slot



Seamless Steel Tubing - $\frac{3}{4}$ " O.D. - 0.120" Wall Thickness
Slots - $\frac{3}{8}$ " Pitch - $\frac{3}{16}$ " Deep - $\frac{1}{16}$ " Wide
2 Groups, 180° Apart - 2 x Full Scale

$I \approx 0.00053 \text{ in.}^4$ Rect. Mom. of Inertia

$J \approx 0.0056 \text{ in.}^4$ Polar " " "

FRAME

TITLE

ORDER

Details of Corrugated
Section for Pipe Model Testing

DATE 9-23-41

DR. H. B. S. & Co.

CALC. 18413, P-2

BOOK M. E. Harvey

NO.

Send registered, deliver to addressee only, return receipt requested.

Mr. La B C. Andrews
~~Stella Andrews~~
Piping Design Consultant
Stella, Nebraska

Alternate address: COM W. Kellogg Co.

Dear Mr. Andrews:

Since it is important that I receive from you an answer to the questions I will ask later in this letter and since my earlier communications have evidently failed to reach you, I am taking the liberty of sending this letter by registered mail, deliver to addressee only, return receipt requested. Since my letters have failed to reach you, it is quite possible that other correspondence and requests for your consulting service have likewise failed to reach you and it is likely that you may wish to take steps to assure that you are not missing profitable business correspondence which may be lost through clerical error or failure someplace along the line.

I am writing a thesis at the U. S. Naval Postgraduate School on the use of model test methods for analysing piping flexibility, making use of a procedure described by Lt. C. F. Rauch, Jr., in his thesis, "U. S. Naval Postgraduate School, Monterey, California, June 1957". My own purpose has been to explore methods of simulating the extra flexibility which short-radius thin-wall elbows introduce into a piping configuration. Your own contribution to the ~~Wiley~~ Wiley-Kellogg book "Design of Piping Systems" refers on page 201 - to "extra flexibility".

If you feel that you can reveal the nature of these devices, I would greatly appreciate learning whatever you are to communicate. If the devices are patented, I would appreciate learning the patent numbers. If there is any published literature describing them, I would appreciate references to it. If you feel that you cannot reveal the nature or details of the devices, I would appreciate receiving whatever information you are prepared to furnish you clients to assure them that the devices do actually perform the function they are supposed to perform together with calibration curves or error estimates, etc.

This problem of simulating the extra flexibility of elbows is certainly a troublesome one and it is my own intention to attempt to handle it by actual use of thin wall curved tubing. I should like to be able to discuss my thesis previous attempts, successful and unsuccessful, to simulate this extra flexibility and certainly your ~~own contribution~~ own contribution deserves to be mentioned in as much detail as your reply to me will permit.

Mr. Peter H. Freeman, U.S.N.,
Box 1455
U.S. Naval Postgraduate School,
Monterey, California,
December 13, 1957.

Gentlemen:

In order to carry out certain experimental work, I am attempting to find suitable thin-walled metal tubing of small diameter. I would appreciate your forwarding me a list of your tubing products having outside diameters of one-quarter inch or less. Please furnish data on wall thicknesses and tolerances, and state prices, including any discounts for volume sales.

The work I am contemplating may involve making bends of extremely short radii, and I would very much appreciate any information you can give me regarding techniques involved in making such bends.

Yours Very Truly,

PETER H. FREEMAN

Tube Turns

A DIVISION OF NATIONAL CYLINDER GAS COMPANY

GENERAL OFFICES
LOUISVILLE 1, KENTUCKY

PRODUCT ENGINEERING AND RESEARCH

November 18, 1957

Lt. Peter H. Freeman, U.S.N.
U.S. Naval Post Graduate School
P.O. Box 1465
Monterey, California

Dear Lt. Freeman:

I have your letter of November 2, which was accompanied by a note from Professor Brock, and would very much like to help you with your problem, but am afraid that an accurate solution is just not in the cards with wire models; at least I cannot think of any way of accomplishing your object, despite the fact that I have been aware of the problem ever since The M. W. Kellogg Company first went into model testing. However, as I recall, they were able to procure thin tubing with which they could simulate elbow properties, at least in certain ranges, although very little of this type of model testing was carried out because of the relatively large amount of time consumed in making models.

As an alternate, you may want to consider one of the following two approximate approaches:

1. Reduce the wire size at the elbows and bends so that the moment of inertia becomes $3/(1.3 + 2k)$ times that of the straight portions. This will produce about the same degree of approximation as the analytical method advanced in Tube Turns Piping Engineering Paper 4.05, to which I assume you have access through Professor Brock; if not, I will be glad to see that our Advertising Department sends you a copy.
2. Make two model tests, one with a model of constant wire size which would underestimate elbow flexibility, and a second with the wire size reduced at the elbows to give $1/k$ times the moment of inertia which would result in an overestimate. In the first model, the elbows would have the proper torsional flexibility, in the second, the proper bending flexibility. Your true result would be bracketed between the two, and usually be somewhat closer to the results of the second test.

Finally, in answer to your direct question, we have no unpublished information pertinent to your problem.

I assume you are aware of the attached paper by two of my colleagues. It demonstrates, both analytically and experimentally, that pressure has the effect of reducing elbow flexibility, particularly in relatively

Tube Turns

Lt. Peter H. Freeman

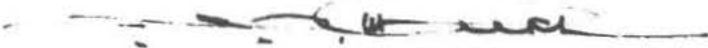
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November 18, 1957

thin-wall assemblies where flexibility factors attain high values. As far as you are concerned, this means a reduction in the range of error in relation to the true behavior of a system under pressure resulting from the use of wire models or approximations of the type I have suggested.

Very truly yours,

T U B E T U R N S



A. R. C. Markl
Research Consultant

ARCM:rre
Attachment

cc: Professor John E. Brock

THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS

29 WEST 39TH STREET • NEW YORK 18, N. Y.

paper no.

56-SA-50



EFFECT OF INTERNAL PRESSURE ON THE FLEXIBILITY AND STRESS INTENSIFICATION FACTORS OF CURVED PIPE OR WELDING ELBOWS

E. C. Rodabaugh, Associate Member ASME
Senior Research Engineer

and

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Louisville 1, Ky.

Contributed by the Power Division for presentation at the ASME Semi-Annual Meeting, Cleveland, Ohio - June 17-21, 1956. (Manuscript received at ASME Headquarters April 2, 1956.)

Written discussion on this paper will be accepted up to July 23, 1956.

(Copies will be available until April 1, 1957)

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Decision on publication of this paper in an ASME journal had not been taken when this pamphlet was prepared. Discussion

ABSTRACT

The flexibility and stress-intensification factors presently applied in piping flexibility analysis to account for the behavior of curved pipe in bending have been derived from theories and tests with no internal pressure. Pressure tends to reduce the effect of these factors but in smaller and relatively thick-wall piping, commonly used in the past, the effect is of a low order and may be neglected; in larger diameter relatively thin-wall piping the effect is pronounced and significant.

Using strain-energy methods, the present paper develops a theory establishing the flexibility and stresses due to in-plane and out-of-plane bending including the effect of internal pressure, and proves its adequacy by means of carefully conducted tests. In a final step, the complex theoretical formulas are reduced to a simple and readily usable approximation.

EFFECT OF INTERNAL PRESSURE ON THE FLEXIBILITY AND STRESS INTENSIFICATION FACTORS OF CURVED PIPE OR WELDING ELBOWS

By E. C. Rodabaugh and H. H. George

It is generally known that curved pipe subjected to bending is more flexible and has higher stresses than would be indicated by the elementary theory of bending. These characteristics of curved pipe are recognized in piping system flexibility calculations by the use of "flexibility factors" and "stress intensification factors" which are simply the ratios of actual flexibility and stress to those predicted by the elementary bending theory.

Under the present rules of the American Standard Code for Pressure Piping (ASA B31.1-1955, Par. 621d) the flexibility factor and stress intensification factor are given by the formulas:

$$k = \frac{1.65}{h}, \geq 1 \quad \dots \dots \dots [1]$$

$$i = \frac{0.9}{h^{1/3}}, \geq 1 \quad \dots \dots \dots [2]$$

where

k = flexibility factor

i = stress intensification factor

h = flexibility characteristic = $\frac{tR}{r^3} = \left(\frac{t}{r}\right)\left(\frac{R}{r}\right)$

t = pipe wall thickness, in.

R = bend radius of curved pipe, in.

r = mean cross section radius of curved pipe, in.

The above formulas are based on theories and tests without internal pressure; however, consideration has been given in the past to the effect of internal pressure on these factors but it was concluded that the effect was negligible. For

relatively thick-wall pipe used at rather low stresses, this is true; however, the recent trend has been towards the increasing use of thin-wall pipe at high stresses and in such piping the effect of the internal pressure becomes significant.

In the present paper, theoretical formulas including the effect of internal pressure are developed for flexibility and stress intensification factors; the development is given in Appendix I. In addition, considerable experimental work has been done to substantiate the theoretical developments and to conclusively demonstrate that internal pressure has a pronounced effect on the flexibility and stress intensification factors; this is described in Appendix II.

The theoretical formulas developed in Appendix I are extremely complex and not easily used by the practical piping engineer. It was found, however, that these complex theoretical formulas could be adequately approximated by the following readily usable formulas:

$$k_p = \frac{k}{1 + \frac{S}{E} \chi_k}, \quad S \geq 1 \quad \dots \dots \dots [3]$$

$$i_p = \frac{i}{1 + \frac{S}{E} \chi_i}, \quad S \geq 1 \quad \dots \dots \dots [4]$$

where

k_p = flexibility factor with internal pressure

i_p = stress intensification factor with internal pressure

(Note: This factor does not include the stress caused directly by internal pressure; it covers that due to bending only)

$S = P r/t$ = stress due to internal pressure in straight pipe, psi

P = internal pressure, psi

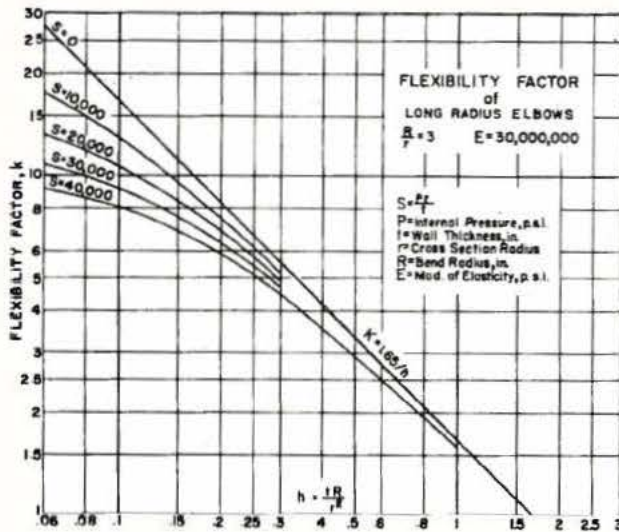


Fig. 1 Effect of pressure on flexibility factors

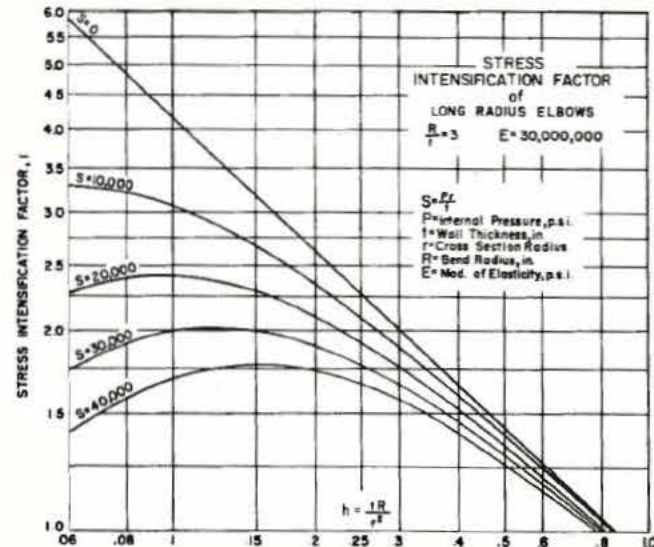


Fig. 2 Effect of pressure on stress-intensification factors

E = modulus of elasticity of pipe material, psi

K_k = function of r/t and $R/r = 6 \left(\frac{r}{t} \right)^{\frac{4}{3}} \left(\frac{R}{r} \right)^{\frac{1}{3}}$

K_1 = function of r/t and $R/r = 3.25 \left(\frac{r}{t} \right)^{\frac{2}{3}} \left(\frac{R}{r} \right)^{\frac{1}{3}}$

The effect of internal pressure on reducing the flexibility and stress intensification factors on a curved pipe is illustrated in Figures 1 and 2, respectively. The example covers long-radius welding elbows ($R/r = 3$) of r/t -ratios ranging from 3 to 50 and the range of internal pressures which produce up to 40,000 psi stress in steel pipe. When h is small the effect of pressure is significant. For example, without internal pressure a 24" standard weight long-radius elbow ($h = .094$) has a flexibility factor of 17.5; with internal pressure corresponding to a stress of 10,000 psi the flexibility factor drops to 13.3, and at 40,000 stress to 8.2. The stress intensification factor drops from 4.3 ($S = 0$) to 3.1 ($S = 10,000$) and to 1.7 ($S = 40,000$).

It is quite understandable that the effect of the internal pressure on thick-wall curved pipe was overlooked because, for

$$\left[\frac{R}{t} \right]$$

$$20 \text{---} 2$$

Example.

In using the nomograph, the left hand scale should be moved to the right approximately two inches. When published in Transactions the nomograph will be reproduced to correct scale.

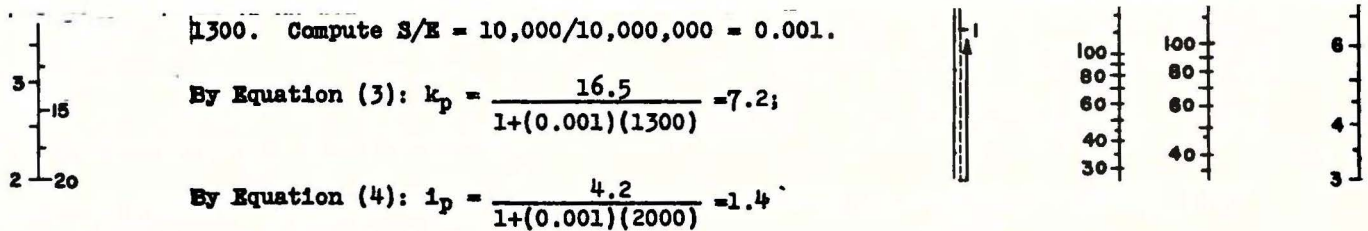


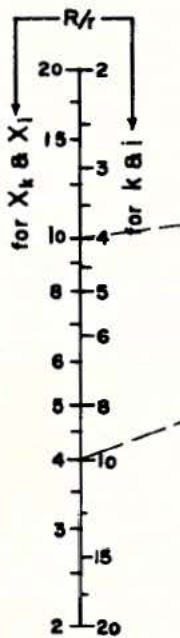
Fig.3 Nomograph for obtaining k , i , k_p and i_p

values of the characteristic h covered by thick-wall pipe the change in flexibility and stress intensification with usual pressures is of the same order as might occur due to commercial variation in pipe wall thickness.

To provide the engineer with a means for readily calculating the factors, a simple nomograph is given in Figure 3 for obtaining k , i , X_k and X_i as defined in formulas [1], [2], [3] and [4], respectively.

Acknowledgement:

The writers wish to gratefully acknowledge their indebtedness to Mr. A. R. C. Markl for encouragement and suggestions.

**Example:**

What are the flexibility and stress-intensification factors of an aluminum ($E = 10,000,000$) curved pipe with $r/t = 40$, $R/r = 4$ at (a) zero pressure; (b) a pressure of 250 psi corresponding to $S = Pr/t = 10,000$ psi?

Solution:

(a) Zero pressure, connect 4 on right side R/r scale with 40 on r/t scale. Read $i = 4.2$, $k = 16.5$.

(b) 250 psi, connect 4 on left side R/r scale with 40 on r/t scale. Read $X_1 = 2000$ and $X_k = 1300$. Compute $S/E = 10,000/10,000,000 = 0.001$.

By Equation (3): $k_p = \frac{16.5}{1+(0.001)(1300)} = 7.2$;

By Equation (4): $i_p = \frac{4.2}{1+(0.001)(2000)} = 1.4$

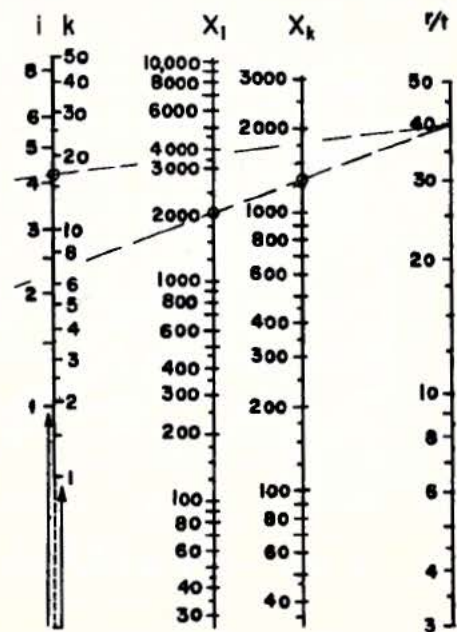


Fig.3 Nomograph for obtaining k , i , k_p and i_p

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APPENDIX I

THEORETICAL DEVELOPMENT

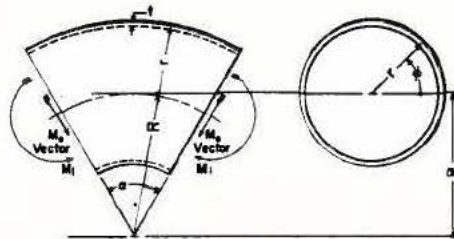


Fig.4 Illustrating the nomenclature

Nomenclature

r = mean cross section radius of curved pipe, in.

t = pipe wall thickness, in.

R = bend radius of curved pipe, in.

I = moment of inertia of pipe cross section $= \pi r^3 t$, in⁴.

E = modulus of elasticity, psi

ν = Poisson's ratio

M = applied moment, in-lb (M_1 , in-plane; M_2 , out-of-plane)

U = energy per unit length of centerline of curved pipe, lb

P = internal pressure, psi

(A subscript P denotes the value of a factor with internal pressure)

S = circumferential stress in straight pipe due to internal pressure $= Pr/t$, psi

α = pipe bend arc, radians

ϕ = circumferential location angle

w = displacement of pipe wall, in. (w_r , radial; w_t , tangential)

ρ = radius of curvature, in a plane perpendicular to a plane containing the bend radius, caused by an out-of-plane bending moment

$\eta = \Delta\alpha/\alpha$ for in-plane bending, $\eta = R/\rho$ for out-of-plane bending

ΔA = increase in pipe cross sectional area, in.²

$\lambda = tR/r^2 \sqrt{1-\nu^2} = h/\sqrt{1-\nu^2}$, flexibility characteristic

$\psi = PR^2/Ert = SR^2/Er^2$, parameter related to pressure

k - flexibility factor

i - stress intensification factor based on bending fatigue tests

ϵ - strain, (ϵ_l , longitudinal, ϵ_c , circumferential)

S_m - stress due to applied moment, psi (S_{ml} , longitudinal; S_{mc} , circumferential)

γ - maximum circumferential stress intensification factor (γ_1 , in-plane; γ_o , out-of-plane)

β - maximum longitudinal stress intensification factor (β_1 , in-plane; β_o , out-of-plane)

Introduction

Theoretical studies of the flexibility and stress intensification factors of curved pipe began some 45 years ago with the work of Th. von Karman (1)⁺, who developed theoretical formulas for in-plane bending of curved pipe without internal pressure. Vigness (2) developed a similar theory for the case of out-of-plane bending. A large amount of experimental work has been done which confirms their theoretical developments.

In 1947 Barthélemy (3) developed the theory for in-plane bending with internal pressure. This paper however, apparently escaped attention in this country. Clark, Gilroy and Reissner (4) developed a theory from which the pressure effect could be deduced. Kafka and Dunn (5) have directly developed the theory and have run confirming experiments.

None of the existing developments cover the case of out-of-plane bending nor have they developed simplified formulas which make the theory readily useable by piping engineers.

⁺Numbers in parenthesis refer to list of references following Appendix II.

In the following, the theory is developed for both in-plane and out-of-plane bending. It is then shown that the complex theoretical formulas can be represented by comparatively simple formulas with adequate accuracy for purposes of practical piping stress analysis.

Basic Equations and Development

The effect of internal pressure can be obtained by a relatively simple extension of the energy methods used by von Karman and Vigness. Accordingly, we start with the basic energy equations developed by these authors.

IN-PLANE BENDING

$$U_i = \frac{t^2 E}{2R} \left\{ \int_0^{2\pi} \left(\frac{r \Delta \alpha}{\alpha} \sin \phi + w_t \cos \phi + w_r \sin \phi \right)^2 d\phi + \frac{t^2 R^2}{12 F^4 (1-\nu^2)} \int_0^{2\pi} \left(\frac{d^3 w_t}{d\phi^3} + \frac{d w_r}{d\phi} \right)^2 d\phi \right\} \quad [5]$$

OUT-OF-PLANE BENDING

$$U_i = \frac{t^2 E}{2R^2} \left\{ \int_0^{2\pi} \left(\frac{r R}{\rho} \cos \phi + w_t \cos \phi + w_r \sin \phi \right)^2 d\phi + \frac{t^2 R^2}{12 F^4 (1-\nu^2)} \int_0^{2\pi} \left(\frac{d^3 w_t}{d\phi^3} + \frac{d w_r}{d\phi} \right)^2 d\phi \right\} \quad [6]$$

Equations [5] and [6] give the elastic energy stored in a unit centerline length of curved pipe due to tangential displacements w_r and radial displacements w_t . The assumption made for in-plane bending is:

$$w_t = \sum_{n=1}^{\infty} a_n \sin 2n\phi \dots \dots \dots [7]$$

and for out-of-plane bending:

$$w_t = \sum_{n=1}^{\infty} b_n \cos 2n\phi \dots \dots \dots [8]$$

With the further assumption of inextensibility in the transverse direction, which implies that $w_r = -dw_t/d\phi$, and

substituting the trigometric series expressions for w_t , equations

[5] and [6] become:

IN-PLANE BENDING

$$U_1 = \frac{r t F}{2 R^2} \left\{ \int_0^{2\pi} \left(\frac{r \Delta \alpha}{\alpha} \sin \phi + \cos \phi \sum_{n=1}^{\infty} a_n \sin 2n\phi - \sin \phi \sum_{n=1}^{\infty} 2n a_n \cos 2n\phi \right)^2 d\phi \right. \\ \left. + \frac{\Delta^2}{12} \int_0^{2\pi} \left(- \sum_{n=1}^{\infty} 8n^3 a_n \cos 2n\phi + \sum_{n=1}^{\infty} 2n a_n \cos 2n\phi \right)^2 d\phi \right\} \dots [9]$$

OUT-OF-PLANE BENDING

$$U_1 = \frac{r t F}{2 R^2} \left\{ \int_0^{2\pi} \left(\frac{r R}{\rho} \cos \phi + \cos \phi \sum_{n=1}^{\infty} b_n \cos 2n\phi + \sin \phi \sum_{n=1}^{\infty} 2n b_n \sin 2n\phi \right)^2 d\phi \right. \\ \left. + \frac{\Delta^2}{12} \int_0^{2\pi} \left(\sum_{n=1}^{\infty} (2n)^3 b_n \sin 2n\phi - \sum_{n=1}^{\infty} 2n b_n \sin 2n\phi \right)^2 d\phi \right\} \dots [10]$$

The first integral in equations [9] and [10] may be put in Fourier series form by use of trigonometric equivalents of the type:

$$\cos \phi \sum_{n=1}^{\infty} a_n \sin 2n\phi = \frac{a_1}{2} \sin \phi + \frac{1}{2} \sum_{n=1}^{\infty} (a_n + a_{n+1}) \sin (2n+1)\phi$$

Performing the indicated integrations in equations [9] and [10], it follows that both equations [9] and [10] become:

$$U_1 = \frac{\pi r t F}{2 R^2} \left\{ r^2 \eta^2 + 3r\eta c_1 + \frac{9}{4} c_1^2 + \frac{1}{4} \sum_{n=1}^{\infty} c_n^2 (1-2n)^2 - 2c_n c_{n+1} (2n-1)(2n+3) \right. \\ \left. + c_{n+1}^2 (2n+3)^2 + \frac{\Delta^2}{12} \sum_{n=1}^{\infty} c_n^2 (8n^3 - 2n)^2 \right\} \dots [11]$$

where $c_n = a_n = b_n$; $\eta = \Delta \alpha / \alpha$ for in-plane bending;

$\eta = R/\rho$ for out-of-plane bending.

When internal pressure is considered, the additional work represented by the internal pressure P acting against the change in volume must be considered; per unit length of centerline, this is:

$$U_2 = P \Delta A \dots [12]$$

ΔA , the change in area of the curved pipe cross section

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$$\Delta A = \frac{1}{2} \int_0^{2\pi} (r + w_r)^2 d\phi - \frac{1}{2} \int_0^{2\pi} r^2 d\phi = \frac{1}{2} \int_0^{2\pi} (2rw_r + w_r^2) d\phi \dots \dots \dots [13]$$

The relation $w_r = -dw_t/d\phi$ is derived from the condition of inextensibility, dropping second and higher order terms. In the case of determining ΔA , however, this is not sufficiently accurate, since the work done by the pressure is itself dependent upon second and higher order terms in ΔA . Considering second order terms, it can be shown that:

$$\Delta A = -2\pi \sum_{n=1}^{\infty} n^2(4n^2-1) c_n^2 \dots \dots \dots [14]$$

In order to determine the values of the coefficients c_n , we may differentiate $U = U_1 - U_2$ with respect to each c_n and, by the principle of "least work", each of the resulting expressions may be set equal to zero; thereby obtaining:

$$\left. \begin{aligned} \frac{\partial U}{\partial c_1} = 0 &= 3r\eta + (5 + 6\lambda^2 + 24\psi) c_1 - \frac{5}{2} c_2 \dots \dots \dots \\ \frac{\partial U}{\partial c_2} = 0 &= -\frac{5}{2} c_1 + (17 + 600\lambda^2 + 480\psi) c_2 - \frac{21}{2} c_3 \dots \dots \dots \\ \frac{\partial U}{\partial c_n} = 0 &= -c_{n-1} \frac{(2n-3)(2n+1)}{2} + c_n \left\{ (4n^2+1) + (8n^3-2n)^2 \frac{\lambda^2}{6} + [8n^2(4n^2-1)] \psi \right\} \\ &\quad - c_{n+1} \frac{(2n-1)(2n+3)}{2} \dots \dots \dots \end{aligned} \right\} \dots [15]$$

Equations [15] give a set of n linear equations with $(n+1)$ unknown c 's. By assuming that $c_{n+1} = 0$, all constants may be evaluated. Since the c 's all contain a factor of $r\eta$, it is convenient to introduce the relation $d_n = c_n/r\eta$.

The minimized energy U is then equated to work done by the bending moment:

$$U_{min} = \frac{1}{2} \frac{M\eta}{R} \dots \dots \dots [16]$$

Flexibility Factor:

Substituting the values of $d_n = c_n/r\eta$ from equations [15] in equation [16] and solving for η :

$$\eta = \frac{RM}{IE} \left[1 + 3d_1 + \frac{3}{4}d_1^2 + \frac{1}{4} \sum_{n=1}^{\infty} d_n^2 (1-2n)^2 - 2d_n d_{n+1} (2n-1)(2n+3) + d_{n+1}^2 (2n+3)^2 \right. \\ \left. + \frac{\lambda^2}{12} \sum_{n=1}^{\infty} d_n^2 (8n^3 - 2n)^2 + \psi \sum_{n=1}^{\infty} 4n^2 (4n^2 - 1) d_n^2 \right]^{-1} \dots \dots \dots [17]$$

The factor in brackets in equation [17] is the generalized formula for the flexibility factor k for both in-plane and out-of-plane bending. It may be simplified to the form:

$$k_p = \frac{1}{1 + \frac{1}{2}d_1} \dots \dots \dots [18]$$

In the first approximation, for example, with c_2 and higher c_n assumed zero, equation [15] gives:

$$d_1 = \frac{c_1}{r\eta} = - \frac{3}{5 + 6\lambda^2 + 24\psi} \dots \dots \dots [19]$$

and from equation [18]:

$$k_p = \frac{1}{1 - \frac{3}{2} \left(\frac{1}{5 + 6\lambda^2 + 24\psi} \right)} = \frac{5 + 6\lambda^2 + 24\psi}{0.5 + 6\lambda^2 + 24\psi} \dots \dots \dots [20]$$

When $P = 0$, equation [20] reduces to the equations derived by von Karman (in-plane bending) and Vigness (out-of-plane bending). Equation [20], when $P \neq 0$, is confirmed by the work of Kafka and Dunn for in-plane bending by substituting $\psi = 0$

in their equations. Note however, that the development herein includes the flexibility factor with internal pressure for out-of-plane bending. It turns out that, as in the case with zero pressure, the flexibility factor for in-plane bending is the same as for out-of-plane bending.

Stresses:

The longitudinal strains on which equations [5] and [6] are based are:

IN-PLANE BENDING

$$e_x = \frac{1}{R} \left(\frac{\Delta \alpha}{\alpha} r \sin \phi + w_t \cos \phi + w_r \sin \phi \right) \dots \dots \dots [21]$$

OUT-OF-PLANE BENDING

$$e_x = \frac{1}{R} \left(\frac{Rr}{\rho} \cos \phi + w_t \cos \phi + w_r \sin \phi \right) \dots \dots \dots [22]$$

Circumferential strains at the outer and inner wall surface, for both in-plane and out-of-plane bending, are given by:

$$e_c = \pm \frac{t}{2(1-\nu^2)r} \left(\frac{d^3 w_t}{d\phi^3} + \frac{d w_r}{d\phi} \right) \dots \dots \dots [23]$$

The plus sign applies to the outside wall surface; minus to inside wall surface.

Strains may be converted to stresses by the usual formulas:

$$S_{mx} = \frac{E}{1-\nu^2} (e_x + \nu e_c) \dots \dots \dots [24]$$

$$S_{mc} = \frac{E}{1-\nu^2} (e_c + \nu e_x) \dots \dots \dots [25]$$

Combining equations [21] through [25] along with equations [7] and [8] and noting that $\eta = R\mu k/IE$, the following equations for the stresses are obtained:

IN-PLANE BENDING

$$S_{mL} = \frac{KM_0 r}{I(1-\nu^2)} \left\{ \left(1 + \frac{3d}{2}\right) \sin \phi + \frac{1}{2} \sum_{n=1}^{\infty} \{d_n(1-2n) + d_{n+1}(2n+3)\} \sin(2n+1)\phi \pm \frac{\nu \lambda}{2} \sum_{n=1}^{\infty} d_n(2n-2n) \cos 2n\phi \right\} \quad [26]$$

$$S_{mC} = \frac{KM_0 r}{I(1-\nu^2)} \left\{ \pm \frac{\lambda}{2} \sum_{n=1}^{\infty} d_n(2n-2n) \cos 2n\phi + \nu \left(1 + \frac{3d}{2}\right) \sin \phi + \frac{\nu}{2} \sum_{n=1}^{\infty} \{d_n(1-2n) + d_{n+1}(2n+3)\} \sin(2n+1)\phi \right\} \quad [27]$$

OUT-OF-PLANE BENDING

$$S_{mL} = \frac{KM_0 r}{I(1-\nu^2)} \left\{ \left(1 + \frac{3d}{2}\right) \cos \phi + \frac{1}{2} \sum_{n=1}^{\infty} \{d_n(1-2n) + d_{n+1}(2n+3)\} \cos(2n+1)\phi \pm \frac{\nu \lambda}{2} \sum_{n=1}^{\infty} d_n(8n^2-2n) \sin 2n\phi \right\} \quad [28]$$

$$S_{mC} = \frac{KM_0 r}{I(1-\nu^2)} \left\{ \pm \frac{\lambda}{2} \sum_{n=1}^{\infty} d_n(8n^2-2n) \sin 2n\phi + \nu \left(1 + \frac{3d}{2}\right) \cos \phi + \frac{\nu}{2} \sum_{n=1}^{\infty} \{d_n(1-2n) + d_{n+1}(2n+3)\} \cos(2n+1)\phi \right\} \quad [29]$$

Stress intensification factors are obtained by dividing equations [26] through [29] by $M_0 r/I$, since these factors are the ratios of the curved pipe stress to the calculated stress by the ordinary beam theory; which stress is simply $M_0 r/I$.

For example, the circumferential stress intensification factor for in-plane bending by a first approximation, from equation [27] is:

$$\frac{S_{mC}}{M_0 r/I} = \frac{K_P}{1-\nu^2} \left\{ \mp 3d\lambda \cos 2\phi + \nu \left[\left(1 + \frac{3d}{2}\right) \sin \phi - \frac{d}{2} \sin 3\phi \right] \right\} \dots \dots \dots [30]$$

When $P = 0$, this equation reduces to that obtained by Gross (6),[†] based on von Karman's development.

The longitudinal stress intensification factor for in-plane bending by a first approximation, from equation [26], is:

$$\frac{S_{mL}}{M_0 r/I} = \frac{K_P}{1-\nu^2} \left\{ \mp 3\nu \lambda \cos 2\phi + \left(1 + \frac{3d}{2}\right) \sin \phi - \frac{d}{2} \sin 3\phi \right\} \dots \dots \dots [31]$$

When $P = 0$ this equation reduces to that obtained by

Gross.[†] When $P \neq 0$, $\nu = 0$ and considering the midwall stress only, this equation gives

$$\frac{S_{m1}}{M_1/I} = K_P \left\{ \left(1 + \frac{3d}{2}\right) \sin \phi - \frac{d}{2} \sin 3\phi \right\} \dots \dots \dots [32]$$

Equation [32] is identical to that obtained by Kafka and Dunn with $\nu = 0$.

In this connection it might be noted that a number of previous papers on this subject give longitudinal stresses at the midwall of the pipe only. The longitudinal stresses at the inner and outer wall surface are affected by the circumferential bending stresses as indicated by Fig. 8.

Assumptions of Theory

The theory is that of thin shells, wherein the wall thickness is assumed small compared to the radius of curvature. This assumption is generally true where the flexibility and stress factors are of importance. The theory assumes that the ratio R/r is large compared to unity. This is not true of welding elbows; however it has been shown, both theoretically (7) and experimentally, that even for R/r as small as 2, the theory is still of adequate accuracy.

It is assumed that the curved pipe or elbow cross section is initially round. Slight deviations from roundness have no significant effect on the theory as given herein, however, it might be noted that if a curved pipe or elbow is slightly out of round, application of pressure will produce rotation of the

[†] Except for the factor $(1 - \nu^2)$ which is omitted from von Karman's development.

elbow ends (if an elbow end is free) or end moments (if the elbow ends are restrained).

The results obtained herein theoretically apply to a curved pipe of any bend arc, α . Actually, the end effects of the attached pipe or flanges can have a pronounced effect for small bend arcs combined with small values of R/r , as shown experimentally by Pardue and Vigness (7).

Convergence:

In series type equations such as those given herein, it is necessary to know how many terms in the series must be used in order to obtain satisfactory accuracy. The rapidity of convergence of the series expressions for the various factors is not uniform; convergence is fastest for the longitudinal stresses, followed by the flexibility factor and slowest for the circumferential stresses. Since the circumferential stress is usually higher than the longitudinal stress, the number of terms required to obtain satisfactory accuracy of the circumferential stress governs the degree of approximation required.

In the case of zero pressure, the flexibility and stress intensification factors depend only upon λ . Comparison of values by successive approximations indicates that the following order of approximation is necessary for accuracy within about 10%:

Value of λ	Approximation Order Required
.5 and smaller	1st
.4 to .16	2nd
.12 to .08	3rd
.06 to .04	4th

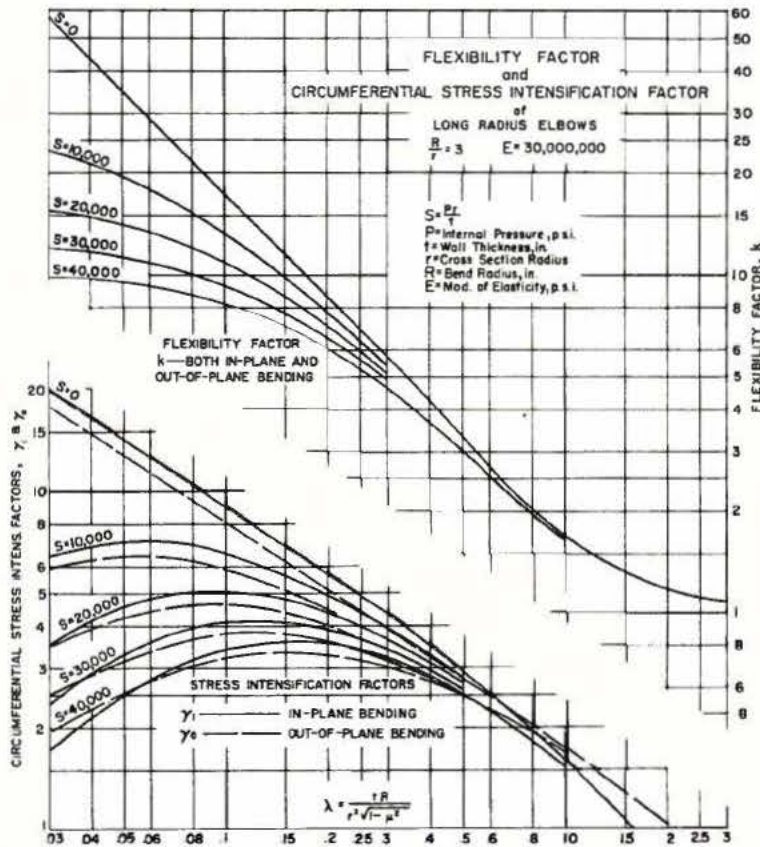


Fig.5 Series solutions for flexibility and stress-intensification factors, long-radius welding elbows

With internal pressure, the series converge somewhat faster than indicated above; however, in general, for thin-wall, short-bend-radius elbows a relatively high order of approximation is necessary for satisfactory accuracy of the circumferential stress intensification factor.

Application of the series formulas to long-radius welding elbows ($R/r = 3$) is shown in Fig. 5. In calculations for Fig. 5, an attempt was made to obtain accuracy within about $\pm 2\%$. It was necessary to use a fifth order approximation for $\lambda = .03$ to attain this accuracy.

Approximate Formulas:

It will be apparent, in writing out equations [18], or [26] through [29], that expressions for the third or fourth-order ap-

proximation become quite lengthy and time consuming to apply.

In the case of zero pressure, it was found graphically by Beskin (8) and later shown analytically by Clark and Reissner (9) that for values of λ less than about .3, the flexibility and maximum stress intensification factors can be expressed by the following simple formulas:

$$K = \frac{1.73}{\lambda} \dots \dots \dots [33]$$

$$\gamma_i = \frac{1.95}{\lambda^{1/3}} \dots \dots \dots [34]$$

$$\gamma_o = \frac{1.67}{\lambda^{1/3}} \dots \dots \dots [35]$$

where k , γ_i , γ_o , are the flexibility factor, in-plane bending circumferential maximum stress intensification factor, and out-of-plane bending maximum circumferential stress intensification factor, respectively; all at zero pressure.[†]

The ASA B31.1 Code formula for k (Equation 1 herein) is simply equation [33] using the parameter $h = tR/r^2$ instead of λ , i.e.:

$$K = \frac{1.73}{\lambda} = \frac{1.73\sqrt{1-\nu^2}}{h} = \frac{1.65}{h}$$

where ν (Poisson's ratio) is taken as 0.3.

The ASA B31.1 Code formula for i (Equation 2 herein) is approximately one-half of equation [34]. This is based on ex-

[†] The maximum longitudinal stress intensification factor is of less practical interest since it generally is smaller than the circumferential factor. For small values of λ , $P = 0$, the ratio of the maximum circumferential stress to the maximum longitudinal stress is about two for in-plane bending and about one and one-half for out-of-plane bending.

tensive bending fatigue tests of elbows (10). It was found that, using the fatigue life of butt welded straight pipe as unity, the effective stress intensification factors of elbows in bending fatigue were about one-half of the theoretical value given by equations [34] and [35].

The fatigue tests showed that the stress intensification factor for in-plane bending was slightly higher than for out-of-plane bending, as indicated by a comparison of equation [34] with equation [35]; however, this difference is rather small for practical significance. Therefore, in the interest of simplicity, the higher i value is given in the Code for application to both in-plane and out-of-plane bending.

The existence of simple asymptotic formulas for the flexibility and maximum stress factors without pressure suggested the possibility of similar formulas for these factors with pressure. Values of k_p , γ_{ip} and γ_{op} were calculated by the series formulas over an appropriate range of the variables S/E , r/t and R/r . In the case of γ_{ip} , the maximum stress occurs at $\phi = 0$. In the case of γ_{op} , the maximum stress lies between 0 and 45° , its value and location was determined by plotting the results of calculations for various values of ϕ in this region. As is the case for zero pressure, the value of γ_{op} was found generally to be less than γ_{ip} . Accordingly, approximate formulas were developed for k_p and γ_{ip} only.

It was found that by plotting the following:

$$\frac{k}{k_p} - / \quad \text{and} \quad \frac{\gamma_i}{\gamma_{ip}} - /$$

against the variables S/E , r/t and R/r on logarithmic coordinates that substantially straight line relations held over the range of these variables likely to be encountered in piping practice, i.e. S/E corresponding to stresses up to 40,000 psi in steel pipe; r/t up to 50 (24" Sch. 10 pipe) and R/r from 2 to ∞ . These plots then led to the approximate formulas;

$$K_p = \frac{K}{1 + 6 \frac{S}{E} \left(\frac{r}{t}\right)^{1/2} \left(\frac{R}{r}\right)^{1/3}} = \frac{K}{1 + \frac{S}{E} X_k} \dots \dots \dots [36]$$

$$\gamma_{ip} = \frac{\gamma_i}{1 + 3.25 \frac{S}{E} \left(\frac{r}{t}\right)^{1/2} \left(\frac{R}{r}\right)^{1/3}} = \frac{\gamma_i}{1 + \frac{S}{E} X_i} \dots \dots \dots [37]$$

These formulas[†] have been found to be within 10% of the theoretical results over the range of variables included where the equations [31], [34] and [35] are themselves accurate. In the regions where the approximate formulas are not in good agreement with theory, the flexibility and stress intensification factors are low and of lesser significance in practical piping applications.

[†] More accurate formulas can be established, however, in such formulas the exponent of S/E , instead of being unity, is a function of r/t . For practical piping work, the slight gain in accuracy is not believed to compensate for the resulting increase in complexity of the formulas.

Appendix II

Comparison of Theory
with Test Data

Like many other developments in the field of theoretical applied mechanics, the theory presented herein is based on a number of simplifying assumptions which enable a relatively simple solution to be obtained. Accordingly, it is desirable to check the theory with tests to make sure that the theoretical development is of adequate accuracy. Numerous investigators have checked theory against test at zero pressure and found adequate agreement. The tests described herein, therefore, are principally concerned with comparing the theoretical pressure effect with experimental results.

Tests were run on a 30" outside diameter, .500" nominal wall welding elbow with a bend radius of 45". The average wall thickness of the elbow was .515" with a variation of $+.058"$, $-.094"$. The outside diameter was 29.973" with a variation of $+.160"$, $-.172"$. The elbow was made of a low alloy, high yield strength proprietary steel.

Each end of the elbow was welded to 59" lengths of 30" outside diameter, .500" nominal wall pipe purchased to API Standard 5LX52. The open ends of these two pipe lengths were closed with 30" welding caps, with a flanged opening in one cap for access to inside strain gages. An overall view of the test assembly is shown in Fig. 6.

In-plane bending moments were applied by means of a tie rod hinged to each cap with a turnbuckle in the tie rod as shown in Fig. 6. Loads were measured by SR-4 strain gages on the tie rod. The rotation of one end of the elbow with re-

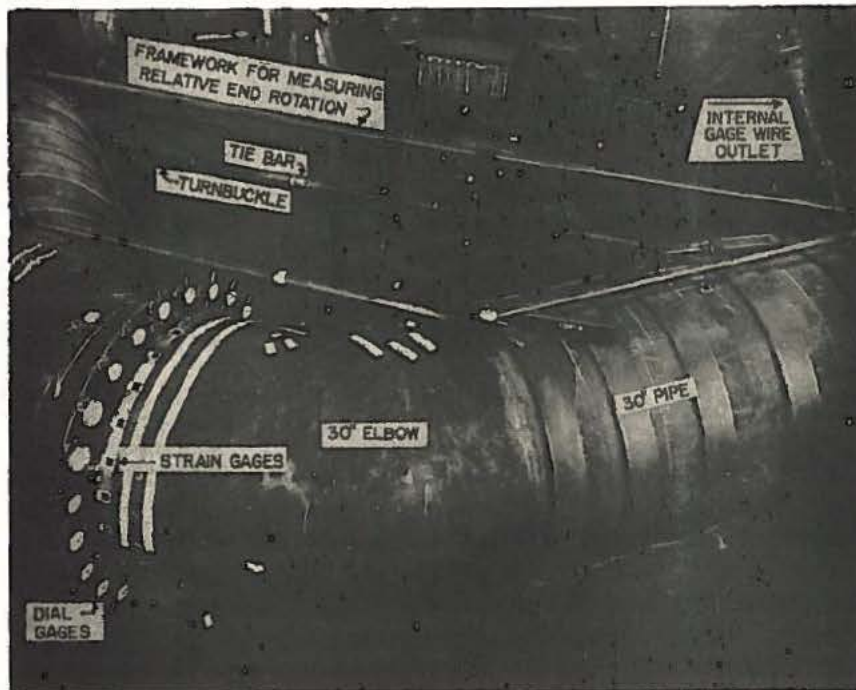


Fig.6 Testing arrangement, 30-in. long-radius welding elbow

spect to the other was measured by means of a framework attached to the straight pipe with dial gages to measure relative movement; the framework can be seen in Fig. 6. In calculating the flexibility factor from these measurements, bending of the 30" straight pipe was taken into consideration.

The stress in the elbow wall was determined by means of SR-4 strain gages placed around the circumference every 15° from crotch ($\phi = -90^\circ$) to back ($\phi = 90^\circ$), plus a check gage on the opposite side ($\phi = 180^\circ$). Strain gages were placed on both the inside and outside wall surfaces.

The tests consisted of applying loads with the turnbuckle in the tie rod in steps up to 30,000 lbs, measuring deformations and strains at each step. This was repeated at internal pressures of 0, 400, 800 and 1100 psi. Several runs were made for each pressure and the results averaged. Pressure was applied with water and, subsequently, with nitrogen gas. Inside

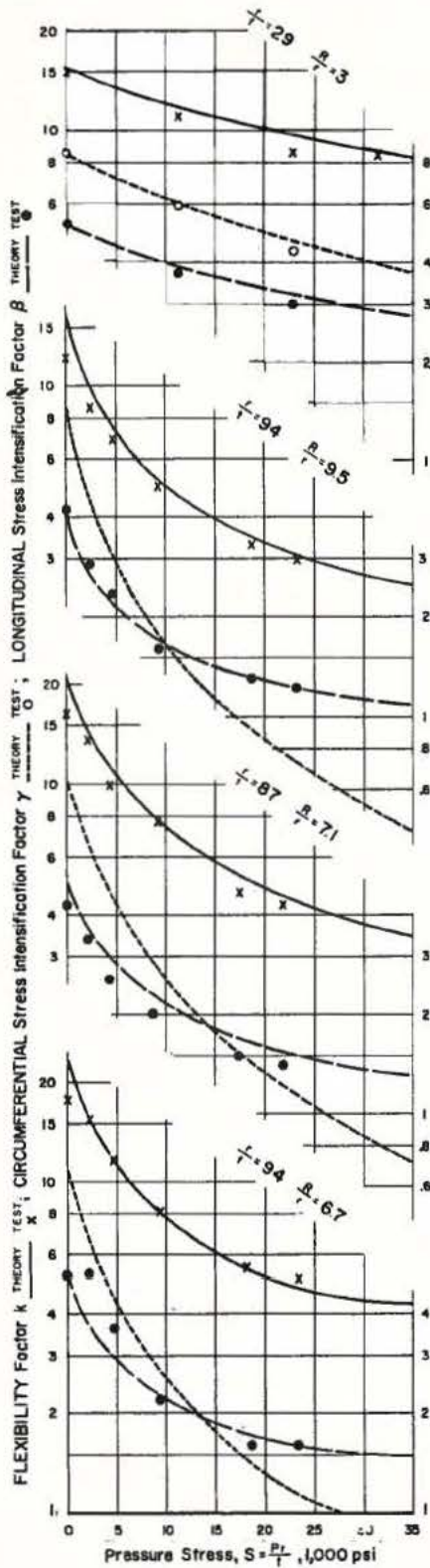


Fig. 7 Change in flexibility factor and maximum stress-intensification factors due to internal pressure

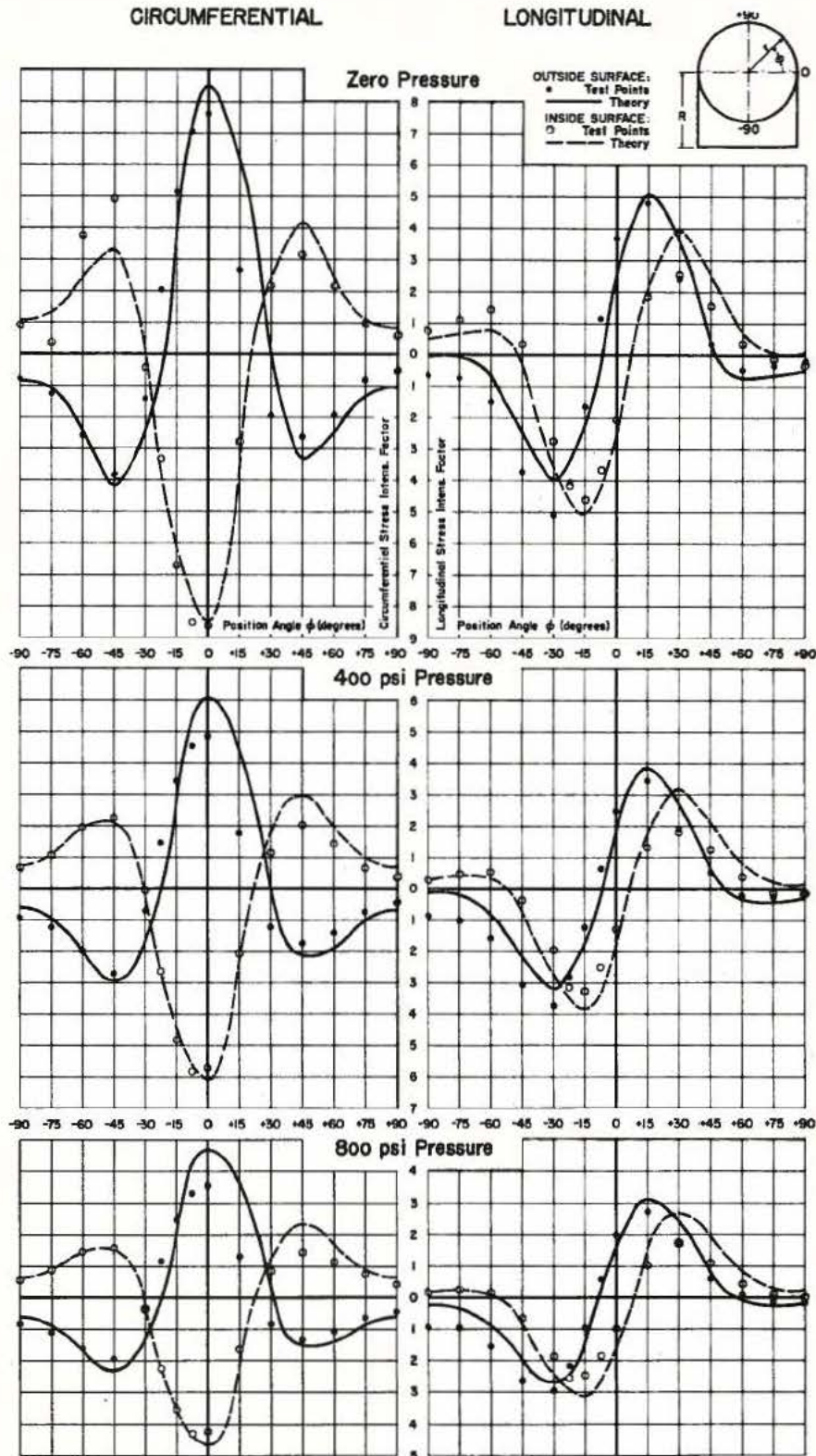


Fig. 8 Variation of stress around circumference of an elbow of 30 in. OD, 0.515 in. average wall, 45 in. bend

strain gages were used with nitrogen gas only.

The flexibility factors and maximum stress intensification factors obtained in these tests are shown in the top graph of Fig. 7. The pattern of stress around the circumference and its variation with pressure is shown in Fig. 8.

Results of similar in-plane bending tests (5) on curved pipe with outside diameters of 3 to $3\frac{1}{2}$ ", wall thicknesses of .016 to .020" and bend radii of 10" to $14\frac{1}{4}$ " are shown in the lower three graphs of Fig. 7. Circumferential stress intensification factors were not reported in these tests.

In general, the theory and tests are in adequate agreement. The fact that the tests and theory do not agree precisely is probably due in part to such experimental factors as non-uniform wall thickness and end effects. In addition, the theory itself is not precise as pointed out in Appendix I.

It is interesting to note that the test results shown cover a wide range of curved pipe dimensions. Agreement of theory and test is adequate for these dimensionally extremely different curved pipes. It is also of interest to note that the approximate equation [33], while not intended to apply for values of r/t much over 50, is still in adequate agreement with theory and tests for the curved pipe where r/t was as high as 94.

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MODEL TEST LABORATORY
POST OFFICE BOX 87
STELLA, NEBRASKA
TELEPHONE: 2651

April 19, 1958

Lt. Peter H. Freeman
Box 1465
U. S. Naval Post Graduate School
Monterey, California

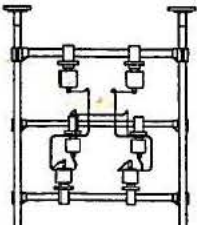
Dear Lt. Freeman:

Please forgive me if I have seemed to neglect you. The past six months or more have been spent in first negotiating for the Model Tester; then dismantling it, and packing and shipping 18,000 pounds of equipment, and re-assembling much of it at this end. As it was all done at my personal expense, it has been necessary to carry on a continuous sales and reorientation effort, as well as to maintain production schedules as far as possible. Probably, at least one of your letters reposes in one of the two hundred or more boxes still unopened in my storeroom.

The vagueness surrounding the description of the device for simulating the flexibility characteristics of curved members reflects the confusion of policy concerning revealment that existed at the time the Piping Manual was written. At that time, the Company was considering the possibility of obtaining new patents on the Model Test equipment, and a lot of experimental work was carried on. It was felt in some quarters that too many details had already been revealed, with the result that Chapter 6 of the Manual has scant resemblance in tone or context to the one I originally wrote.

Now that I am free of company affiliations, the possibility arises of obtaining a patent on the device myself; and, since it is very much my own invention, it seems only fitting that I should at least write the first paper on it. However, if anyone wants to employ the pieces in their own model tests, I see no objection to supplying them on a guaranteed-to-work basis. I do not design them, however, from any theoretical equation, but from my own empirical curves, checking them experimentally before incorporating them in a model.

The problem now is to design one that is adjustable through some reasonable range, so that each one doesn't have to be tailor-made for one specific representation, or, at least, so a fine adjustment can be made. However, when a shop is set up to make them quickly, obtaining different flexibilities is an almost negligible complication; except that, once one is built, it cannot be changed, even slightly.



LAYOUT
INSPECTION

FLEXIBILITY ANALYSIS
BY
MODEL TEST
MANUAL COMPUTATION

RESEARCH
FULL SCALE TESTS

None-the-less, this design represents a reasonably clear-cut answer to the reproduction of the flexibility characteristics of curved tubes in solid rod models, in so far as the theory of this flexibility is valid. Dr. James H. Potter of Stevens Institute doesn't think the presently employed formulas are entirely valid; and I personally feel that a good deal of experimental work is in order all along the line. There is just a little bit of a tendency to over-emphasize what is known, utilizing it somewhat too complacently, while ignoring what is not known; though, of course, all engineering has necessarily to be prosecuted, to some extent, on this thesis.

I am in somewhat of a quandary as to how to reply to your request. Whereas I am quite naturally anxious to have the device mentioned favorably in your own or any paper on the subject, and have no desire to be secretive about it in any way (many people have seen it), I feel that a proper appraisal of it can hardly be expected without a detailed presentation which, recently, I haven't had time to prepare. Then, once the presentation is made, if I want to patent it, I have to do so within a limited time. If I can think of a way to resolve all these problems, I shall be happy to tell you all about it. It is really a very simple thing, easily made, and not nearly as profound a conception as I make it sound. Maybe you can let me know when you plan to complete your thesis.

Very truly yours,


Lale C. Andrews

LCA/av

Ed. Dony
Commission de Recherches
Tuyauteries.
U.E.E.B.
7 Cantersteen
Bruxelles.

Bruxelles.le 1 décembre 57.

Cher Monsieur Freeman,

Je vous remercie de votre lettre.

Je n'ai malheureusement pas grand chose à dire sur le sujet.

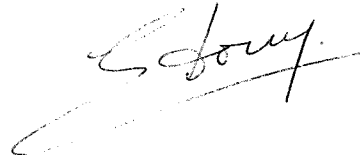
Nous avons traité le problème d'une manière différente, tous nos modèles réduits sont faits entièrement en tubes, aussi bien pour les parties droites que pour les courbes.

Ces tubes sont choisis de telle manière que le rapport:épaisseur, diamètre extérieur soit le même pour le modèle réduit que pour la tuyauterie réelle.

Nous disposons de tubes ayant des épaisseurs de 0,5 1 1,5 2 et 2,5 mm ce qui nous permet d'être très près des coefficients de flexibilité des coudes de la tuyauterie réelle et de ne pas devoir nous occuper de ce problème, en ce qui concerne nos essais sur modèles réduits.

Je reste à votre entière disposition et serai toujours très heureux de correspondre avec vous.

Croyez, cher Monsieur Freeman à mes meilleurs sentiments

A handwritten signature in dark ink, appearing to read 'Ed. Dony', with a long horizontal flourish extending to the left.

Am. Stand
Steel Birth-Wedding
Pittsburgh
ASA 2.16.9-1951